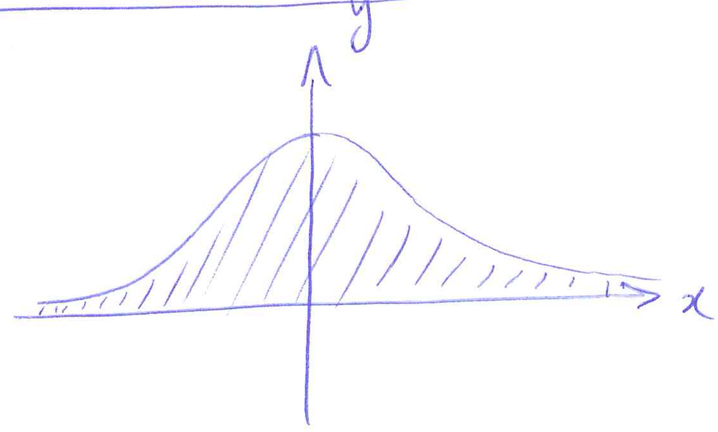
$$\boxed{\int_{-\infty}^{+\infty} e^{-x^2}\,dx = ?}$$

$$b^2 = \underbrace{\int_{-\infty}^{+\infty} e^{-x^2}\,dx}_{b} \cdot \underbrace{\int_{-\infty}^{+\infty} e^{-y^2}\,dy}_{b}$$

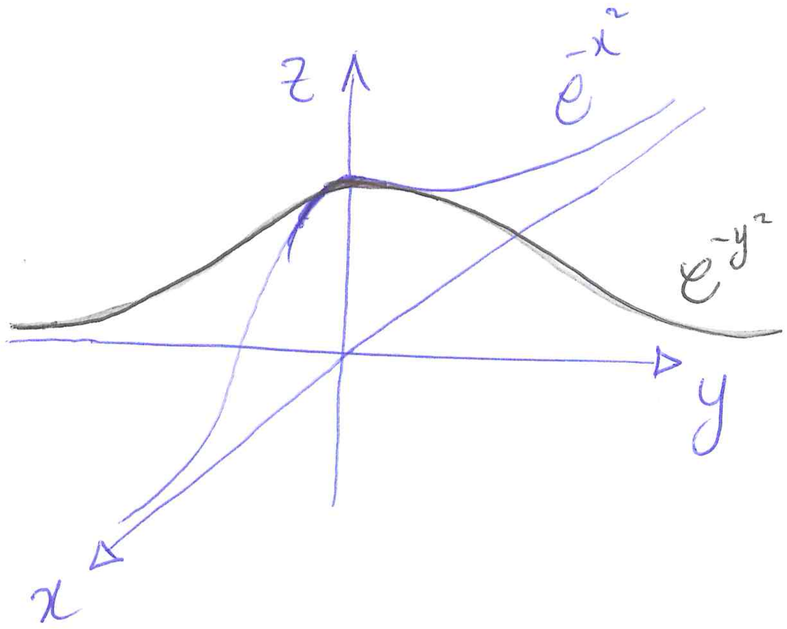
$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-x^2}\cdot e^{-y^2}\,dx\,dy$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-(x^2+y^2)}\,dx\,dy$$

Summer Shahzad



$$\int_{-\infty}^{+\infty} e^{-x^2}\,dx = b > 0$$



- coordinate polari

$$x = r\cos\theta \qquad dx = \left\{ \frac{\partial x}{\partial r}, \frac{\partial x}{\partial \theta} \right\}$$

$$y = r\sin\theta \quad \longrightarrow \quad dy = \left\{ \frac{\partial y}{\partial r}, \frac{\partial y}{\partial \theta} \right\}$$

$$r^2 = x^2 + y^2$$

$$\begin{Bmatrix} dx \\ dy \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \cdot r \\ \sin\theta & r\cos\theta \end{bmatrix} = \underline{\underline{A}}$$

$$\det \underline{\underline{A}} = r\cos^2\theta + r\sin^2\theta = r \cdot 1 = r.$$

$$\Rightarrow \rho^2 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-r^2}\, dx\, dy = \int_0^{2\pi} \int_0^{+\infty} e^{-r^2} \cdot r\, dr\, d\theta$$

$$= 2\pi \cdot \left[-\frac{1}{2} e^{-r^2} \right]_0^{+\infty} = \pi$$

$$p^2 = \pi \longrightarrow \boxed{p = \sqrt{\pi}}$$

$$\boxed{p = \int_{-\infty}^{+\infty} e^{-x^2}\,dx = \sqrt{\pi}}$$

fine!