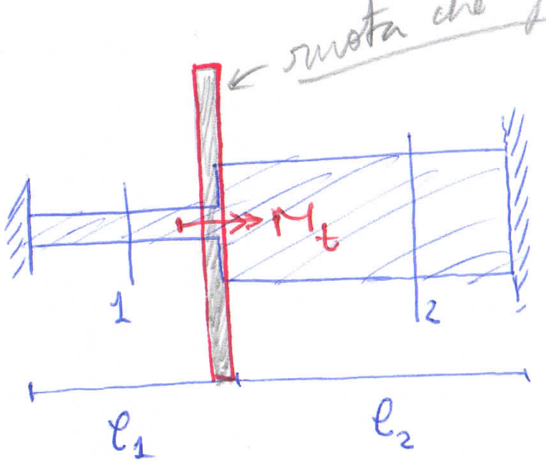
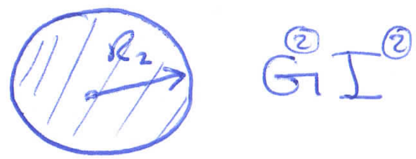
Es. Torsione — Con Th. Menabrea

[illegible]

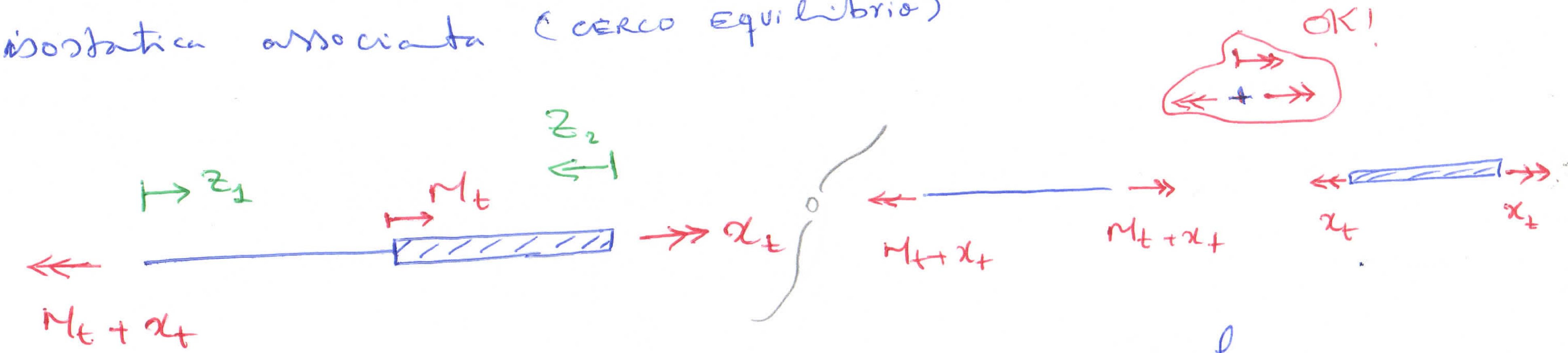


Sezione 1 ; Sezione 2



i) → torsionalmente 1ª volta iperostatica.

ii) → isostatica associata (cerco equilibrio)



iii) Scrivo $\mathcal{E}_{tot.} = \frac{1}{2 I_p^{(1)} G^{(1)}} \int_0^{\ell_1} \left[(M_t + x_t) \right]^2 dz_1 + \frac{1}{2 G^{(2)} I_p^{(2)}} \int_0^{\ell_2} (x_t)^2 \, dz$

$$\xi = \frac{1}{2}\left[\frac{1}{I_p^{(1)} G^{(1)}}(M_t + x_t)^2 \, l_1 + \frac{1}{I_p^{(2)} G^{(2)}} x_t^2 \cdot l_2\right]$$

$$\frac{\partial \xi}{\partial x_t} = \frac{1 \cdot 2}{\cancel{2}}\left[\frac{\cancel{2}(M_t + x_t)\, l_1}{I_p^{(1)} \; G^{(1)}} + \frac{\cancel{2} \cdot x_t}{I_p^{(2)} G^{(2)}} \cdot l_2\right] = 0$$

$$= \left[\frac{M_t + x_t}{\frac{I_p^{(1)} G^{(1)}}{l_1}} \cdot + \frac{x_t}{\frac{I_p^{(2)} \cdot G^{(2)}}{l_2}}\right] = 0$$

$$\Rightarrow \quad x_t\left[\frac{1 \cdot l_1 \cdot \cancel{I_p^{(2)} G^{(2)}}}{\frac{\cancel{I_p^{(1)} G^{(1)}}}{\cancel{l_1}}} + \frac{I_p^{(1)} G^{(1)} \, 1 \cdot l_2}{\frac{I_p^{(2)} G^{(2)}}{\cancel{l_2}}}\right] = - \frac{M_t \cdot l_1}{\frac{\cancel{I_p^{(1)} G^{(1)}}}{\cancel{l_2}}}$$

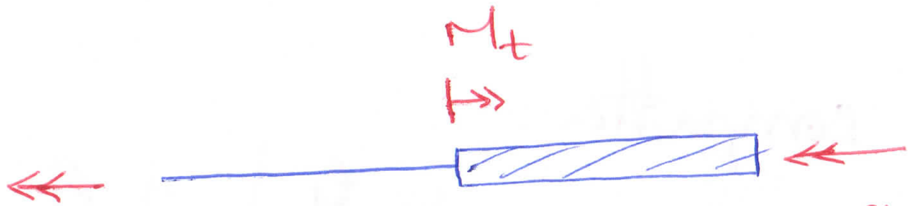
$$x_t \left[1 \cdot l_1 + \frac{l_2 \cdot I_p^{(1)} G^{(1)}}{I_p^{(2)} G^{(2)}} \right] = - M_t \cdot l_1$$

$$x_t \cdot \left[\frac{I_p^{(2)} G^{(2)} \cdot l_1 + l_2 I_p^{(1)} G^{(1)}}{I_p^{(2)} G^{(2)}} \right] = - M_t \cdot l_1$$

$$x_t = \frac{- M_t \cdot l_1 \cdot I_p^{(2)} G^{(2)}}{1 \, I_p^{(2)} G^{(2)} \cdot l_1 + \frac{l_2 I_p^{(2)} G^{(1)}}{l_1 I_p^{(2)} G^{(2)}}}$$

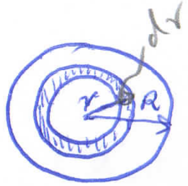
infine

$$x_t = \frac{- M_t}{1 + \frac{l_2}{l_1} \frac{I_p^{(1)} G^{(1)}}{I_p^{(2)} G^{(2)}}}$$



$$\underbrace{M_t - x_t}_{} = \left[\frac{M_t}{1 + \frac{\ell_1}{\ell_2}\frac{G^{(2)} I_p^{(2)}}{G^{(1)} I_p^{(1)}}}\right]$$

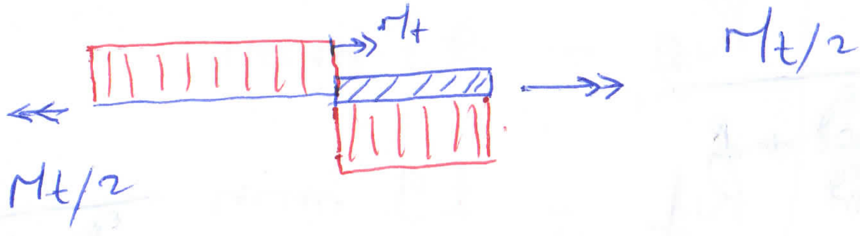
$$x_t = \left[\frac{M_t}{1 + \frac{\ell_2}{\ell_1}\frac{I_p^{(1)} G^{(1)}}{I_p^{(2)} G^{(2)}}}\right]$$



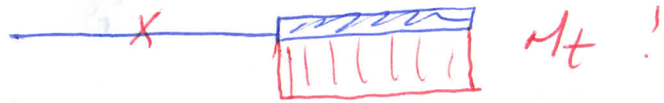
$$I_p = I_x + I_y = \int_A (x^2 + y^2)\, dA = \int_A r^2\, dA = \int_0^R 2\pi r \cdot r^2\, dr = \frac{\pi R^4}{2}$$

Casi Limiti:

2e $\begin{cases} G^{(1)} I_p^{(1)} = G^{(2)} I_p^{(2)} \\ \ell_1 = \ell_2 \end{cases} \Longrightarrow$

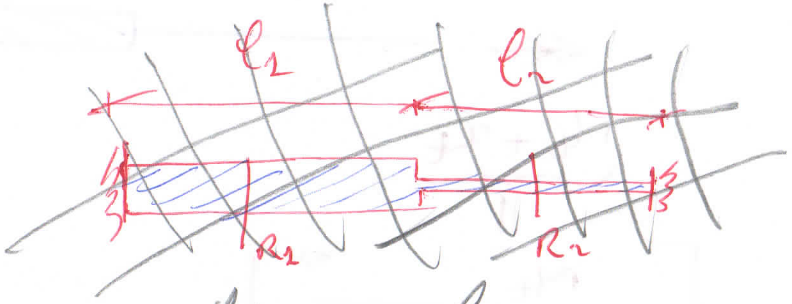


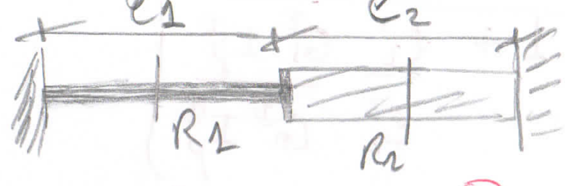
2e $G^{(1)} I_p^{(1)} << G^{(2)} I_p^{(2)} \Rightarrow$



τ nella sezione compatta:

$$\tau^{(1)} = \frac{M_t^{(1)} \cdot R^{(1)}}{I_p^{(1)}}$$





Dat.

$G = 77,2$ GPa (acciaio)

$M_t = 1,4 \cdot 10^6$ N.mm.

$l_2 = 200$ mm

$l_1 = 250$ mm.

$R_2 = 25$ mm

$R_1 = 19$ mm

Ris. ⇒ $\tau^{(2)} = 45,0243$ MPa

$\tau^{(1)} = 27,37$ MPa.

OK!

rif. Mecc. solidi pgg. 155 (McGraw-Hill) Es. 3.45