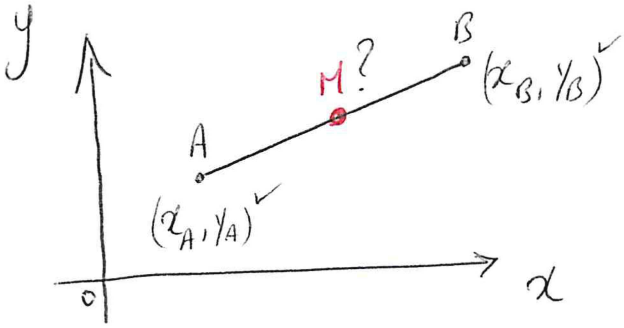
Topografia

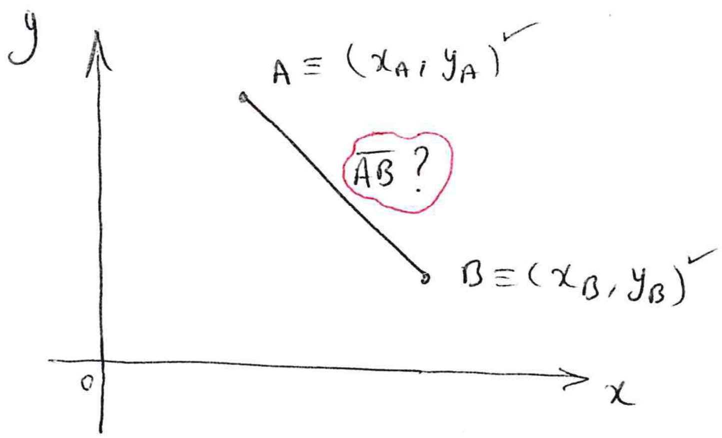
Summer Shahzad

- calcolo punto medio di una retta:



$$\boxed{x_M = \frac{x_A + x_B}{2} \quad ; \quad y_M = \frac{y_A + y_B}{2}}$$

- il Modulo o distanza:



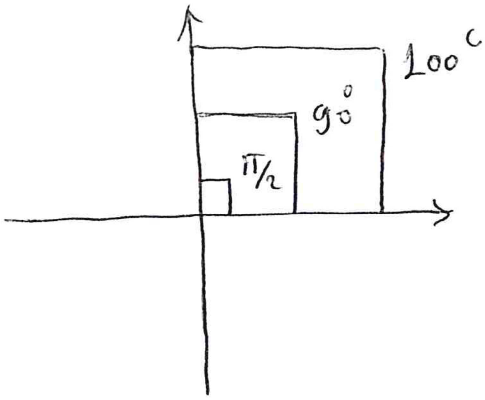
$$\boxed{\overline{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}}$$

- Angoli in generale e nella calcolatrice scientifica:

rad : radianti [e.g. $0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi, 2\pi$]

deg : gradi sessagesimali [$0°, 90°, 180°, 270°, 360°$]

grad : gradi centesimali [$0^c, 100^c, 200^c, 300^c, 400^c$]



- Trasformazione da uno all'altro:

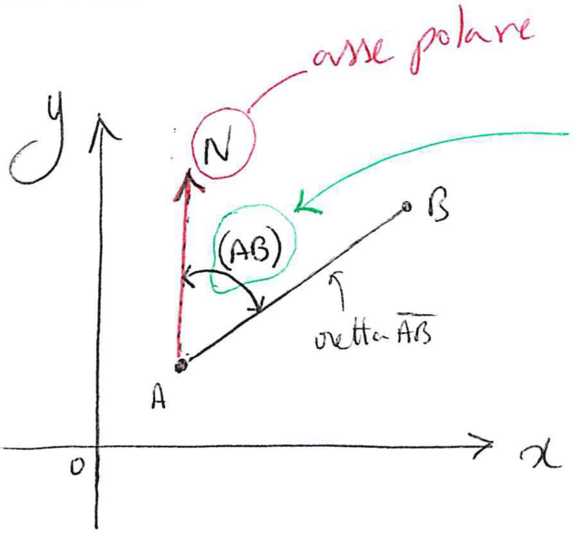
rad → deg : $\frac{3\pi}{4} = \frac{3\pi}{4} \cdot \frac{180°}{\pi} = 135°$

deg → rad : $66° = 66° \cdot \frac{\pi}{180°} = \frac{11}{30}\pi$ rad

deg → grad : $66° = 66° \cdot \frac{100^c}{90°} = \left(\frac{73}{3}\right)^c$

grad → deg : $51^c = 51^c \cdot \frac{90°}{100^c} = 45,9°$

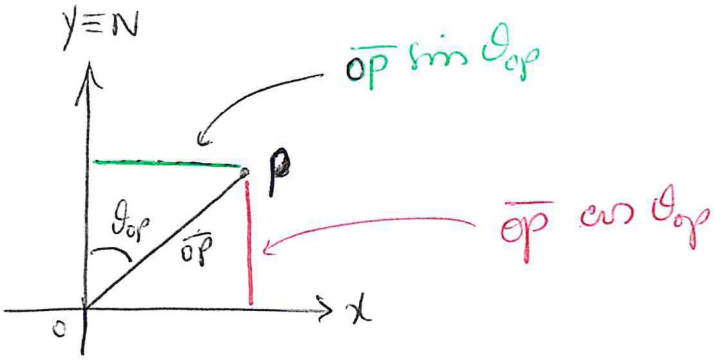
- Nomenclatura:

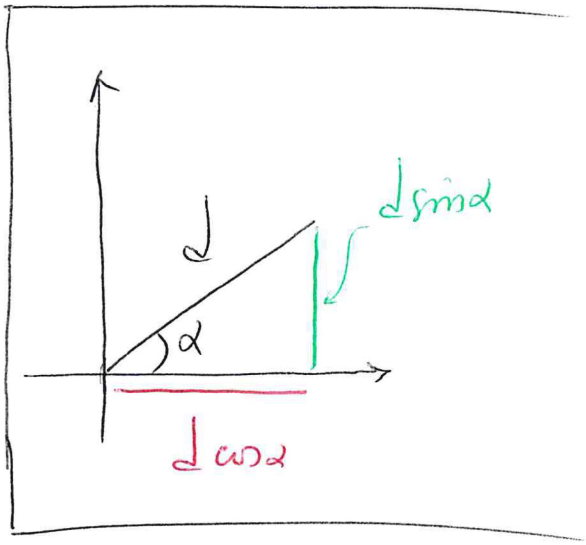


azimut o **angolo di direzione**

si indica anche con

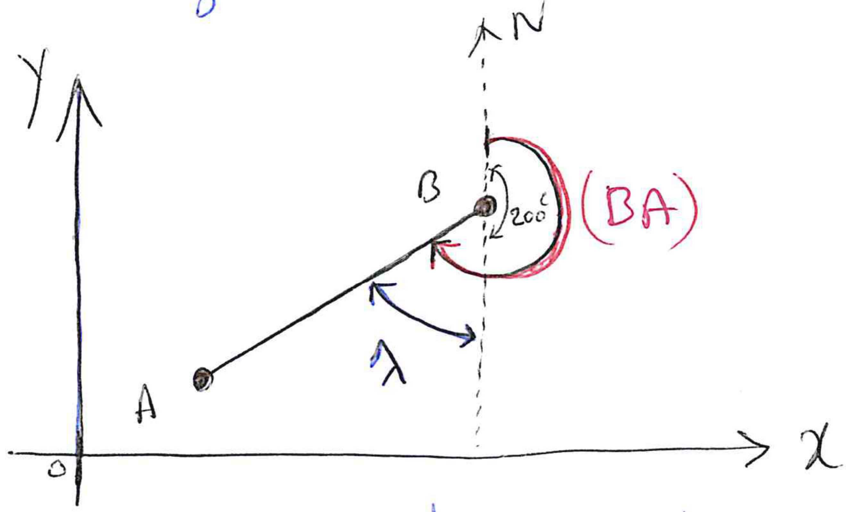
$$\boxed{\theta_{AB} \equiv (AB)}$$





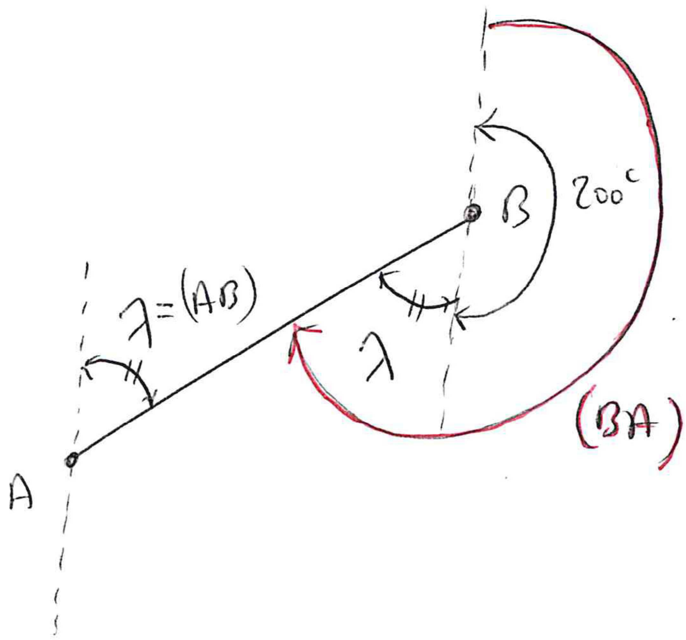
- Trovare l'azimut:

I



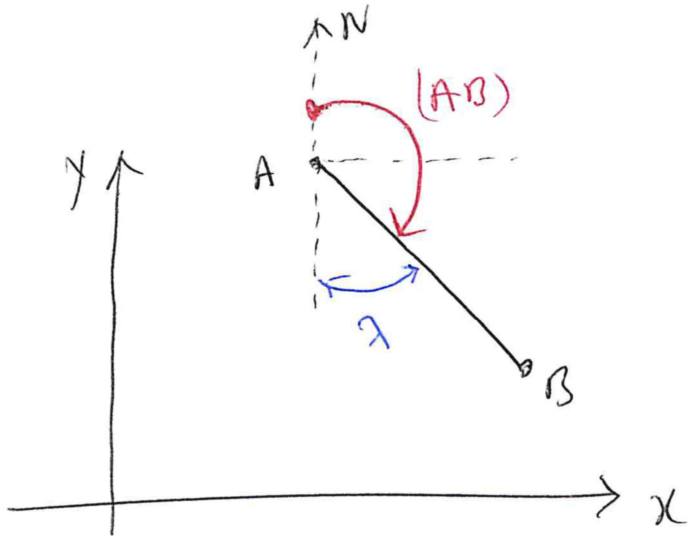
$$\lambda = (BA)^* = \operatorname{arctg}\left|\frac{x_A - x_B}{y_A - y_B}\right|$$

$$\boxed{(BA) = \lambda + 200^c}$$



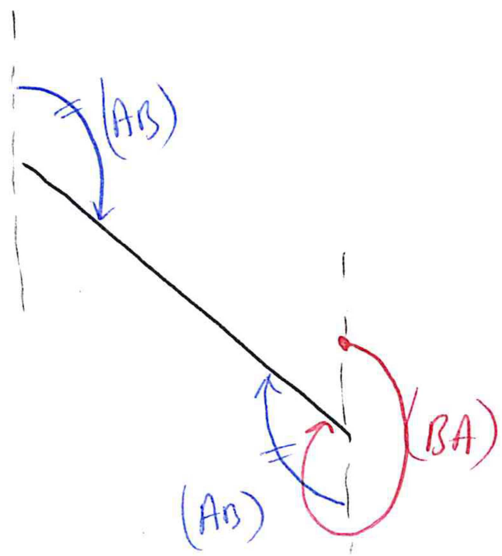
$$\boxed{(AB) = (BA) - 200^c = \lambda}$$

II



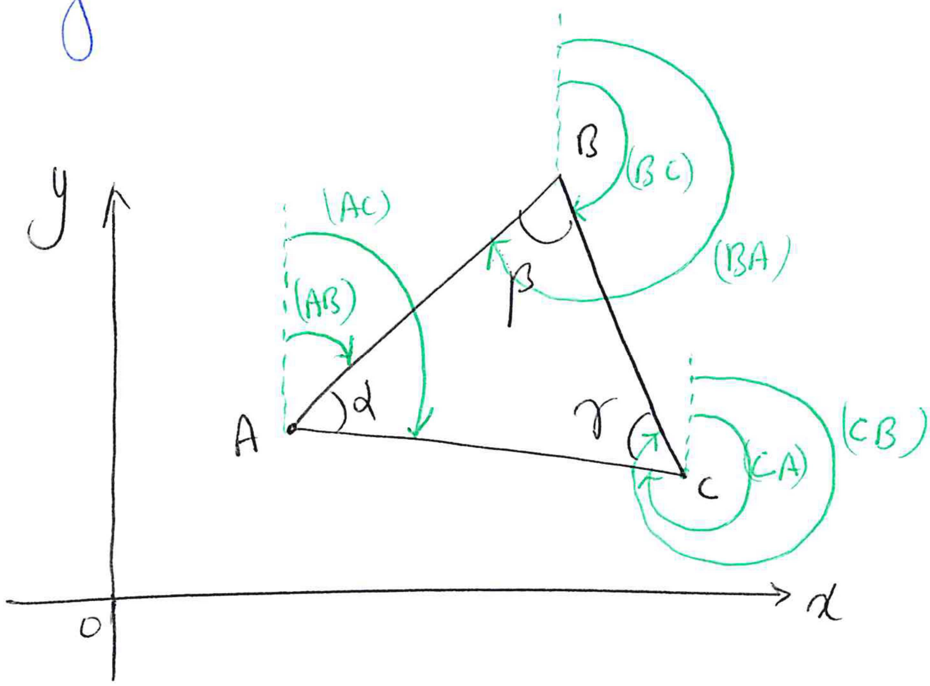
$$\lambda = (AB)^* = \operatorname{arctg}\left|\frac{x_B - x_A}{y_B - y_A}\right|$$

$$\boxed{(AB) = 200^c - \lambda}$$



$$\boxed{(BA) = (AB) + 200^c}$$

⊙ Triangolo



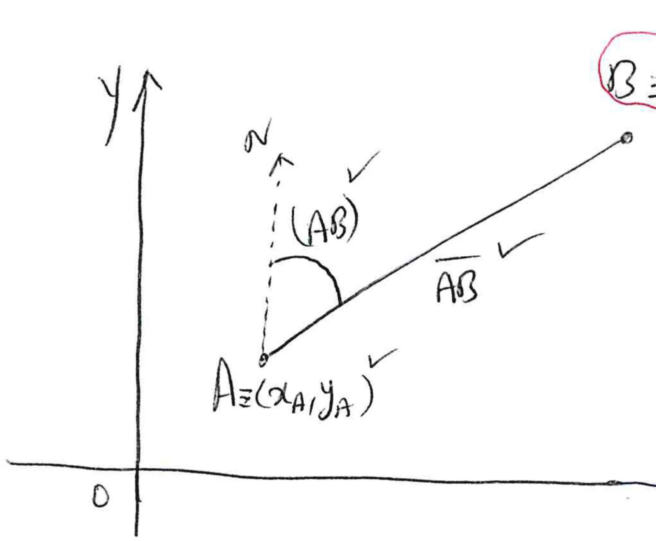
$$\begin{cases} \alpha = (AC) - (AB) \\ \beta = (BA) - (BC) \\ \gamma = (CB) - (CA) \end{cases}$$

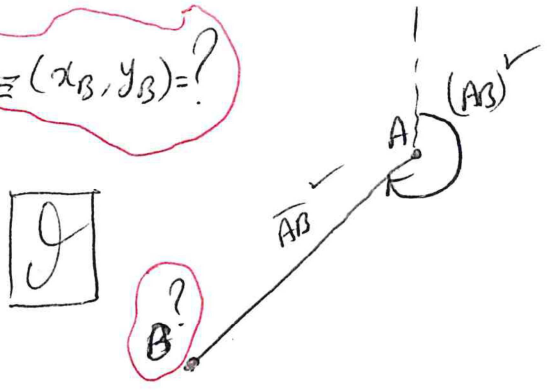
$$(AB) = \operatorname{arctg} \left| \frac{x_B - x_A}{y_B - y_A} \right|$$

$$(BA) = (AB) + 200^c$$

conosco il punto (x_A, y_A) e la distanza $(\overline{AB})$ trovare le coordinate dell'altro punto (x_B, y_B)?

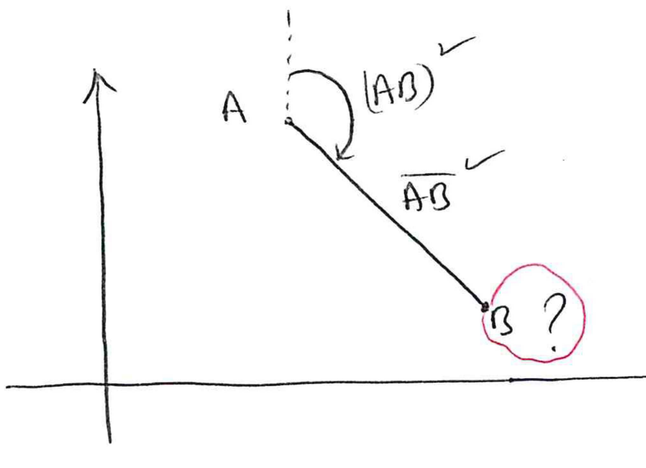
I

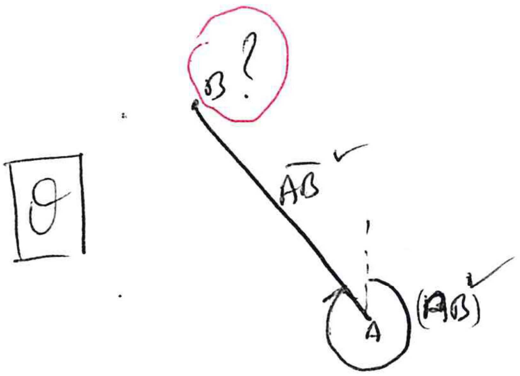




$$\boxed{\begin{aligned} x_B &= x_A + \overline{AB}\,\sin(AB) \\ y_B &= y_A + \overline{AB}\,\cos(AB) \end{aligned}}$$

II





$$\boxed{\begin{aligned} x_B &= x_A + \overline{AB}\,\sin(AB) \\ y_B &= y_A + \overline{AB}\,\cos(AB) \end{aligned}}$$

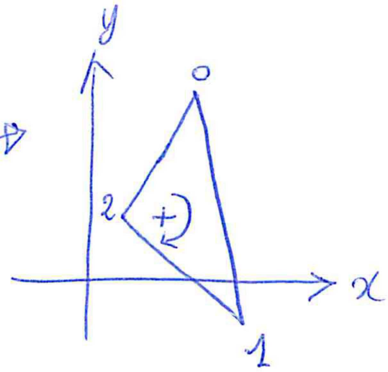
Calcolo Area Poligoni con "n" vertici

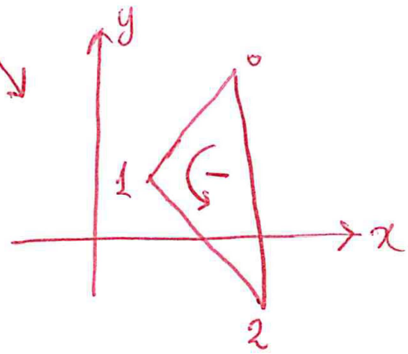
I Coordinate cartesiane

$$A = \pm \frac{1}{2} \sum_{i=1}^{n} y_i \left(x_{i+1} - x_{i-1} \right)$$

or

$$= \pm \frac{1}{2} \sum_{i=1}^{n} x_i \left(y_{i-1} - y_{i+1} \right)$$



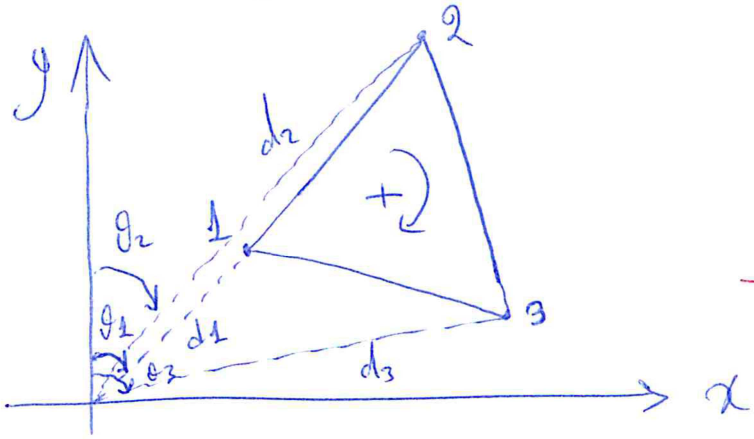


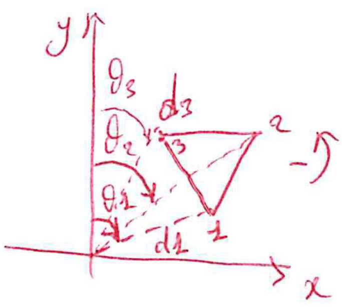
II Coordinate polari

$$A = \pm \frac{1}{2} \sum_{i=1}^{n} d_i d_{i+1} \sin\left(\vartheta_{i+1} - \vartheta_i \right)$$

or

$$= \mp \frac{1}{2} \sum_{i=1}^{n} d_i d_{i+1} \sin\left(\vartheta_i - \vartheta_{i+1} \right)$$



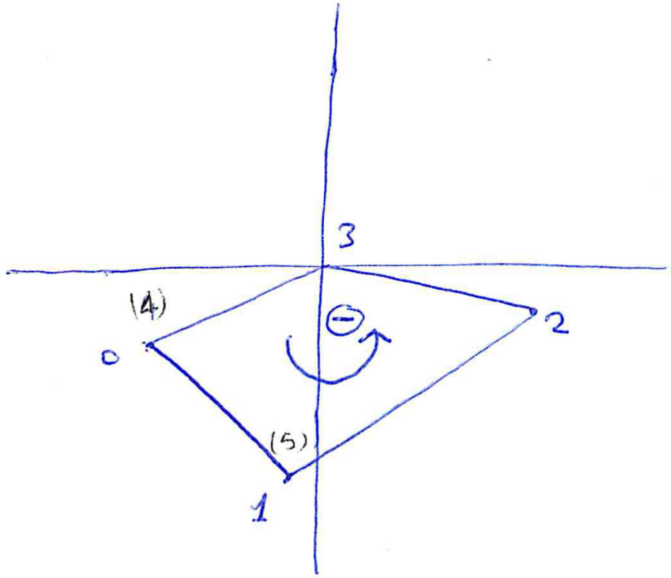


Coordinate cartesiane:

es 1

$\begin{cases} x_0 = -149{,}84 \text{ m} \\ y_0 = -27{,}73 \text{ m} \end{cases}$; $\begin{cases} x_1 = -10{,}87 \text{ m} \\ y_1 = -114{,}61 \text{ m} \end{cases}$; $\begin{cases} x_2 = 117{,}32 \text{ m} \\ y_2 = -92{,}33 \text{ m} \end{cases}$; $\begin{cases} x_3 = 0 \text{ m} \\ y_3 = 0 \text{ m} \end{cases}$

$$A = -\frac{1}{2} \sum_{i=1}^{4} y_i \left(x_{i+1} - x_{i-1} \right)$$



$$= -\frac{1}{2} \left[y_1 (x_2 - x_0) + y_2 (x_3 - x_1) + y_3 (x_4 - x_2) + y_4 (x_5 - x_3) \right]$$

$$= -\frac{1}{2} \left[-114{,}61 (117{,}32 - (-149{,}84)) + (-92{,}33)(0 - (-10{,}87)) + 0 \overset{0}{\cancel{(-149{,}84 - 117{,}32)}} + -27{,}73(-10{,}87 - 0) \right]$$

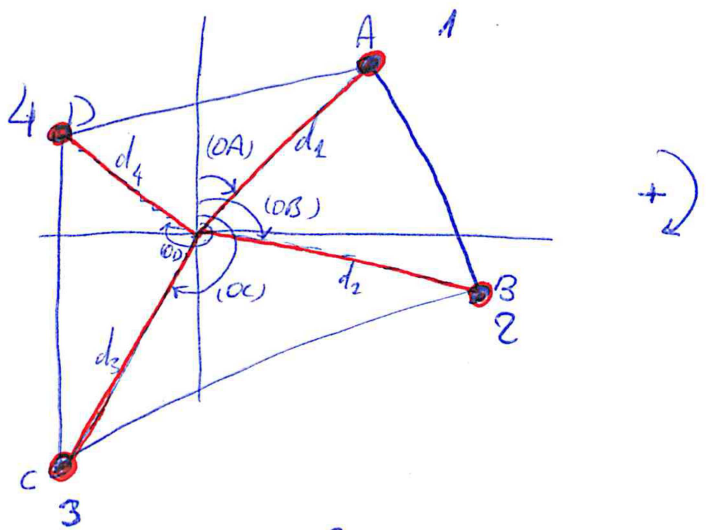
$$= 15660{,}7 \text{ m}^2$$

es 1

1)

es 2) coordinate polari dei vertici

$$\begin{cases} \widehat{(OA)} = 67^c,8710 & \theta_1 = \theta_5 \\ \overline{OA} = 269,27\ m & d_1 \equiv d_5 \end{cases} ; \quad \begin{cases} \widehat{(OB)} = 142^c,2265 & \theta_2 \\ \overline{OB} = 278,38\ m & d_2 \end{cases} ; \quad \begin{cases} \widehat{(OC)} = 242^c,4080 & \theta_3 \\ \overline{OC} = 300,51\ m & d_3 \end{cases} ; \quad \begin{cases} \widehat{(OD)} = 363^c,9411 & \theta_4 \\ \overline{OD} = 239,10\ m & d_4 \end{cases}$$



$A = 142\,328,80\ m^2$

$$A = +\frac{1}{2}\sum_{i=1}^{4} d_i d_{i+1} \sin(\theta_{i+1} - \theta_i)$$

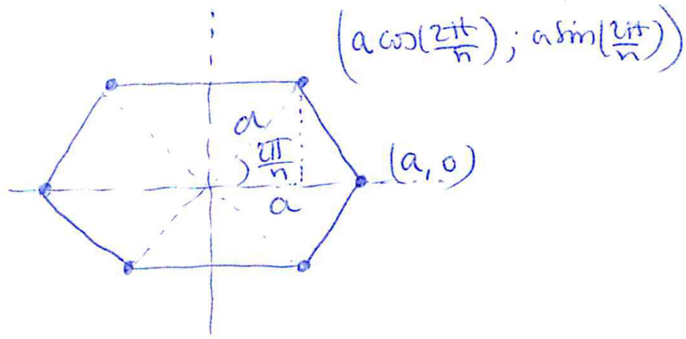
$$= \frac{1}{2}\Big[d_1 d_2 \sin(\theta_2 - \theta_1) + d_2 d_3 \sin(\theta_3 - \theta_2) + d_3 d_4 \sin(\theta_4 - \theta_3) + d_4 d_5 \sin(\underbrace{\theta_5}_{\theta_1} - \theta_4)\Big]$$

$$= \frac{1}{2}\Big[(269,27)(278,38)\sin(142^c,2265 - 67^c,8710) + (278,38)(300,51)\sin(242,4080 - 142^c,2265) + (300,51)(239,10)\sin(363^c,9411 - 242^c,4080) + (239,10)(269,27)\sin(67^c,8710 - 363^c,9411)\Big]$$

$$\cong 142328\ m^2$$

Area di un poligono regolare di "n" vertici o "n" lati:

$\boxed{n = 6}$



$$\boxed{A = n a^2 \sin\left(\frac{2\pi}{n}\right)}$$

⊙ Eq. goniometriche del tipo

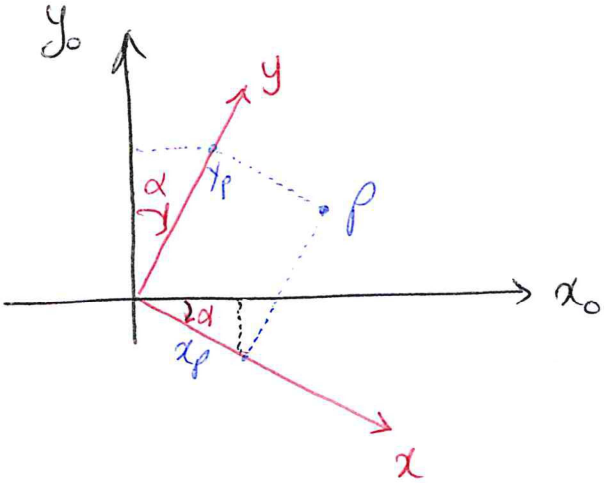
$$\sin(f(x)) = \sin(g(x))$$

$\boxed{\text{I}}$ $\sin(f(x)) = \sin(g(x)) \Rightarrow \begin{cases} f(x) = g(x) + 2k\pi \quad e \\ f(x) = \pi - g(x) + 2k\pi \end{cases}$

$\boxed{\text{II}}$ $\cos(f(x)) = \cos(g(x)) \Rightarrow f(x) = \pm g(x) + 2k\pi$

$\boxed{\text{III}}$ $\tan(f(x)) = \tan(g(x)) \Rightarrow f(x) = g(x) + k\pi$

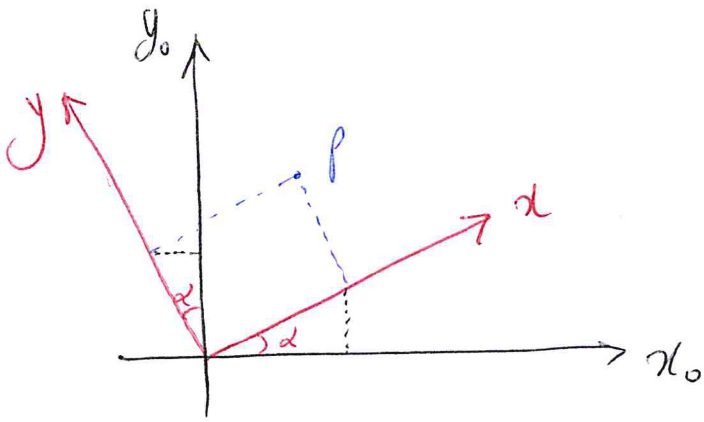
Rotazione degli assi:



$$x_p = x_0 \cos\alpha - y_0 \sin\alpha$$

$$y_p = x_0 \sin\alpha + y_0 \cos\alpha$$

$$\Rightarrow \begin{Bmatrix} x_p \\ y_p \end{Bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix}$$

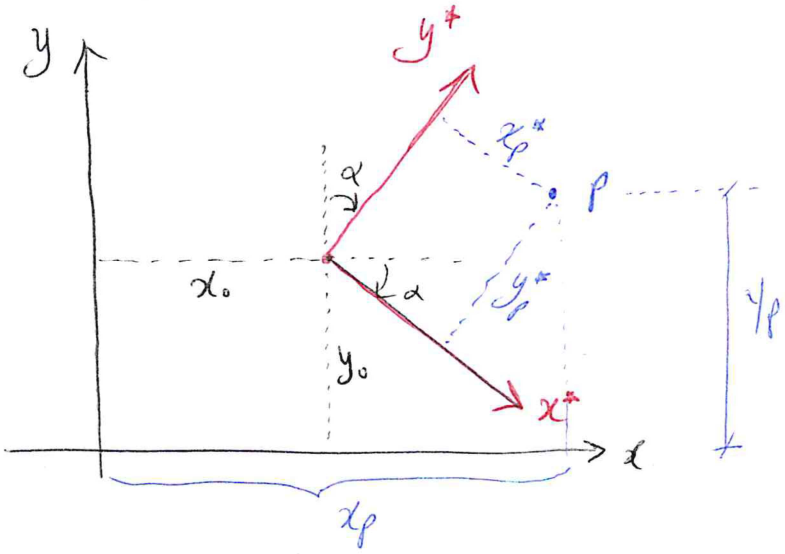


$$x_p = x_0 \cos\alpha + y_0 \sin\alpha$$

$$y_p = -x_0 \sin\alpha + y_0 \cos\alpha$$

$$\Rightarrow \begin{Bmatrix} x_p \\ y_p \end{Bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix}$$

- Rototraslazione:

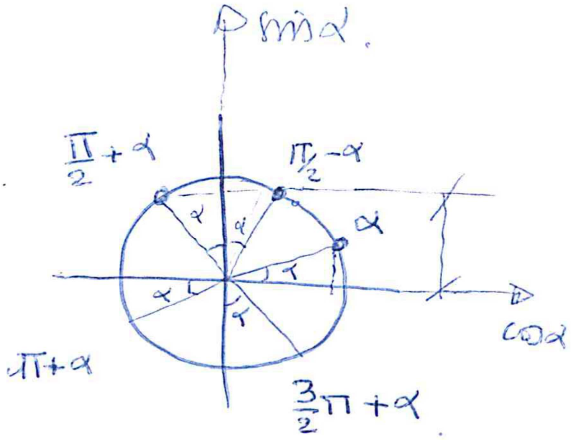


$$
\begin{cases}
x_p^* = (x_p - x_0)\cos\alpha - (y_p - y_0)\sin\alpha \\
y_p^* = (x_p - x_0)\sin\alpha + (y_p - y_0)\cos\alpha
\end{cases}
$$

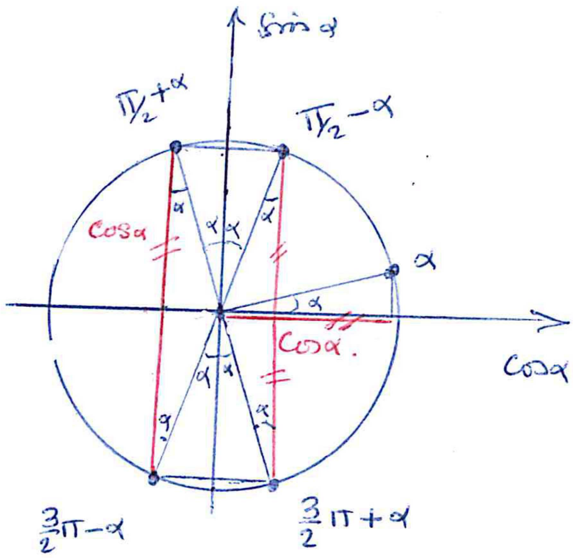
SHAHZAD

SUMMER

$$\begin{cases} \sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha \\ \sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha \\ \sin\left(\frac{3}{2}\pi-\alpha\right) = -\cos\alpha \\ \sin\left(\frac{3}{2}\pi+\alpha\right) = -\cos\alpha \end{cases}$$

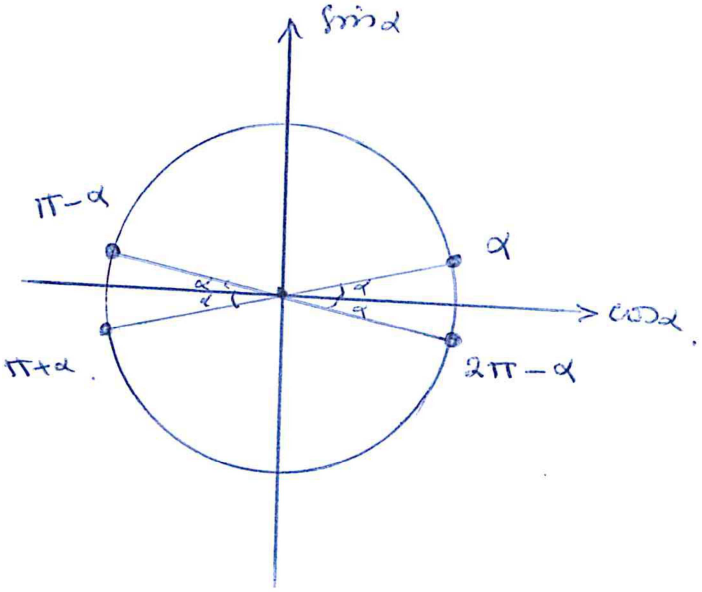


$$\Rightarrow \boxed{\begin{cases} \sin\left(\frac{\pi}{2}\pm\alpha\right) = \cos\alpha \\ \sin\left(\frac{3\pi}{2}\pm\alpha\right) = -\cos\alpha \end{cases}}$$



$$\begin{cases} \cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha \\ \cos\left(\frac{\pi}{2}-\alpha\right) = +\sin\alpha \\ \cos\left(\frac{3}{2}\pi+\alpha\right) = +\sin\alpha \\ \cos\left(\frac{3}{2}\pi-\alpha\right) = -\sin\alpha \end{cases}$$

$$\begin{cases} \sin(\pi-\alpha) = \sin\alpha \\ \sin(\pi+\alpha) = -\sin\alpha \\ \sin(2\pi-\alpha) = -\sin\alpha \\ \sin(2\pi+\alpha) = \sin\alpha \end{cases}$$



$$\begin{cases} \cos(\pi-\alpha) = -\cos\alpha \\ \cos(\pi+\alpha) = -\cos\alpha \\ \cos(2\pi+\alpha) = \cos\alpha \\ \cos(2\pi-\alpha) = \cos\alpha \end{cases}$$