Fibre reinforced concrete (FRC) design

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Abstract

Fibers can be used to improve the behaviour at serviceability limit state (SLS) since they can reduce crack spacing and crack width, thereby improving durability. Fibers can be used to improve the behaviour at ultimate limit state (ULS) where they can partially or totally substitute conventional reinforcement. The mechanical properties of a cementitious matrix are modified when fibers are added. However, elastic properties and compressive strength are not significantly affected by fibers. Generally the compressive relations valid for plain concrete also apply to FRC. While for mechanical behaviour in tension, which is the most important aspect of FRC, three point bending test proposed by the fib Model Code for concrete structures is used to determine the load deflection relation, see [1]. Moreover rigid-plastic model is used for slab design and verification in accordance with [2].

1 FRC according to the CNR § 2.5, page 15 [2]

1. The physical and mechanical properties of the composite are determined by the dosages and by the properties of the individual components (cement matrix and fibers);

2. The addition of fibers can improve the toughness, durability, as well as the impact resistance (resilience), fatigue and abrasion of the cement matrix;

3. The mechanical properties of fiber reinforced concrete must be directly determined on specimens through standardized tests.

4. The minimum dosage of fibers for structural applications must be not less than 0.3% by volume;

5. In the absence of specific tests, all properties that are not specified herein, can be likened to those of ordinary concrete.

2 Mechanical properties of hardened concrete CNR § 2.5.2, page 16 [2]

2.1 Behaviour in compression

- The fibers are generally able to reduce the brittleness of the matrix, but do not significantly affect the compression behavior.
- In practice, the constitutive relations of fiber-reinforced concrete and in particular its resistance can be assimilated to those of the unreinforced material.

2.2 Behaviour in tension

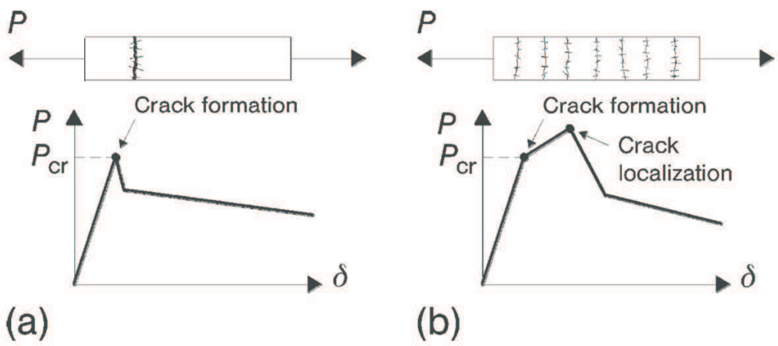

Figure 1: Softening (a) and hardening (b) behaviour in axial tension [1].

- The fibers improve the tensile behavior of the cracked matrix. Depending on their composition, FRC can show hardening or softening behaviour under uniaxial tension, as shown in figure 1;
- For low fiber content (with volume fractions less than 2%) the behavior is softening;
- In the case of high fiber content (with volume fractions higher than 2%), the strength can be higher than that of the matrix, so that a hardening behavior occurs, which is due to the multi-cracking phenomenon, see (b) of figure 1.
- Due to the complexity of phenomenon it is suggested, therefore, a performance based approach that experimentally identifies the constitutive tensile curve through appropriate tests on fiber reinforced concrete specimens. The relationship nominal stress - crack opening, $\sigma_N - w$, can be determined by uniaxial tensile or bending tests. The uniaxial tensile test directly provides the $\sigma_N - w$ relationship and may

be performed in compliance with UNI U73041440. In case of material's softening behavior, this test is not of simple execution. Alternatively, one can use the bending test carried out in accordance with UNI 11039.

- The bending test on un-notched specimen is recommended in case of thin elements subjected to bending, this test allows a better determination of the sectional ductility.

- The post-cracking resistance can be defined based on point values, f_i, corresponding to specified nominal value of crack opening, or average values, f_{eqi}, calculated on an assigned interval of crack opening, see figure 2. In the case of notched specimens, the opening of the crack can be conventionally assumed equal to the displacement between two points at the notch tip, crack tip opening displacement (CTOD).

- The $\sigma_N - w$ relationship deduced from the bending test, is directly applicable to elements subjected to bending. For items subject to tensile load, the resistance by a factor of 0.7 should be penalized. In the case where the result of the bending test on notched specimen is of a hardening type repeat the test on the un-notched specimen, to verify the real ductility in the absence of the notch. In figure 2 f_{eq1} and

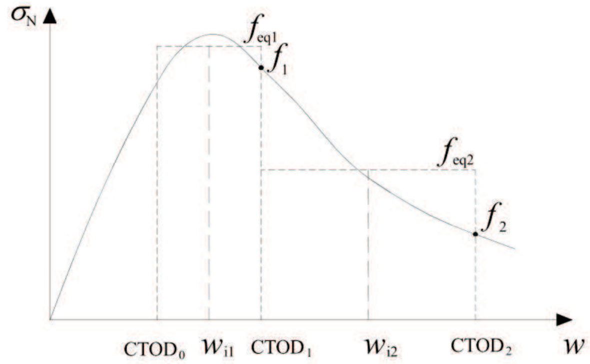

Figure 2: Definition of punctual and average residual strength.

f_{eq2} are, respectively, the post-cracking equivalent resistance for the serviceability limit state and the ultimate limit state (Figure 9-1a);

- Based on data derived from the bending test one can define two simplified relations: tension-crack opening, with rigid-plastic or linear (hardening or softening) post-cracking behavior. In the latter, f_{Fts} represents the serviceability residual strength, defined as the post-cracking strength for serviceability crack opening, and f_{Ftu} represents the ultimate residual strength. The stress values, f_{Fts} and f_{Ftu}, characterizing these two models can be evaluated in the manner specified further.

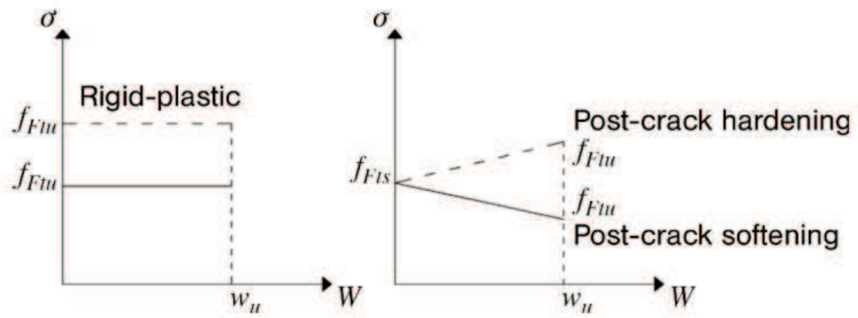

Figure 3: Simplified post-cracking constitutive laws : stress-crack opening (continuous and dashed lines refer to softening and hardening post-cracking behaviour, respectively).

- For the purposes of the definition of the constitutive law of materials with softening behavior, the ultimate value of the crack opening, w_u, may not exceed the maximum of 3 mm, subjected to bending elements, and 1.5 mm in the case of elements subject to tension.
- For materials with hardening behavior, i.e. in the presence of multi-cracking, it is not necessary to determine the opening of the crack opening as it is possible to operate directly in terms of tensions and deformations as specified below.
- In the case of material with hardening behavior, cracking occurs in diffuse manner and, therefore, a mean strain can be adopted directly obtained from the experimental tests for the identification of constitutive parameters. The ultimate strain value assumes 1%.

3 Some restrictions § 3.1, page 23 [2]

- The use of concrete for structural purposes FRC with softening behavior is permitted if the following ratio is satisfied

$$\frac{f_{Ftsk}}{f_{Ftuk}} > 0.2. \tag{1}$$

- In all FRC structures it is necessary to ensure that the following relation is satisfied:

$$\frac{\alpha_u}{\alpha_1} \geq 1.2. \tag{2}$$

where α_u represents the maximum load and α_1 that of first cracking (approximate values of these are shown table 12-1 of Appendix D of [2])

4 Partial safety factors FIB § 5.6.6, page 150 [1]

Design values for the post-cracking strength parameter at ULS can be determined as (see Figure 3):

$$f_{Ftsd} = \frac{f_{Ftsk}}{\gamma_F}, \quad f_{Ftud} = \frac{f_{Ftuk}}{\gamma_F}. \tag{3}$$

The recommended values for the partial safety factors are given in the table (figure 4). For serviceability limit states (SLS), the partial factors should be taken as 1.0.

Material	Partial safety factors
FRC in compression	As plain concrete
FRC in tension (limit of linearity)	As plain concrete
FRC in tension (residual strength)	$\gamma_F = 1.5$

Figure 4: Partial safety factors.

5 Plate elements § 4.3, page 35 [2]

5.1 Elements without conventional reinforcement

For plate elements without conventional reinforcement (figure 5) subject to prevailing bending stress, the verification of resistance may be performed with reference to the resistance moment, m_{Rd}, evaluated assuming **rigid-plastic constitutive law** (figure 3):

$$m_{Rd} = \frac{f_{Ftud} \cdot t^2}{2} \tag{4}$$

where f_{Ftud} is the ultimate residual design strength given in the following paragraphs and t is plate thickness.

In case of simultaneous action of two bending moments m_x and m_y acting in orthogonal directions, the verification of ULS requires that the following relation must be satisfied:

$$\left(\frac{m_x}{m_{Rd}}\right)^2 + \left(\frac{m_y}{m_{Rd}}\right)^2 \leq 1 \tag{5}$$

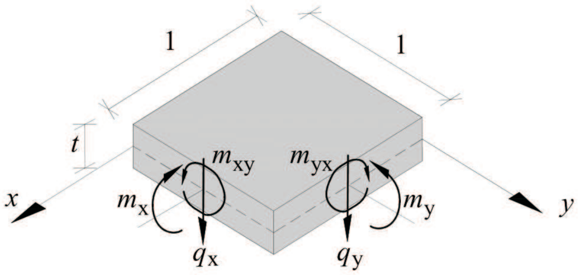

Figure 5: Plate element subject to bending.

5.2 Elements with conventional reinforcement

The verification of fiber-reinforced concrete elements with conventional reinforcement can be performed with the methods traditionally adopted for plain concrete; the contribution of the fibers can be considered adopting nonlinear analysis methods (limit analysis, incremental non-linear analysis).

6 Rigid-plastic model § 5.6, page 145 [1]

The rigid-plastic model takes the static equivalence into account as shown in figure 6, that is f_{Ftu} results from the assumption that the whole compressive force is concentrated in the top fiber of the section.

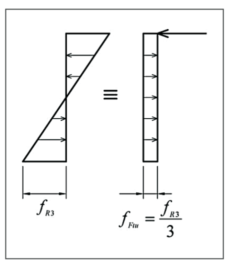

Figure 6: Simplified model adopted to compute the ultimate residual tensile strength in uni-axial tension f_{Ftu} by means of the residual nominal bending strength f_{R3}.

$$M_u = \frac{f_{R3} \cdot b \cdot h_{sp}^2}{6} = \frac{f_{Ftu} \cdot b \cdot h_{sp}^2}{2} \tag{6}$$

where M_u is the ultimate moment, while f_{R3} and f_{Ftu} are described in the folowing paragraph.

The rigid-plastic model identifies an unique reference value, f_{Ftu}, based on the ultimate behaviour. Such a value is determined as:

$$f_{Ftu} = \frac{f_{eq2}}{3} = \frac{f_{R3}}{3}. \tag{7}$$

where f_{R3} represents the residual flexural tensile strength (for f_{eq2}, see figure 2) and can be evaluated from the $f_j - CTOD_j$ relationship given by FIB model page 145 [1];

$$f_{Rj} = \frac{3F_j \cdot l}{2b \cdot h_{sp}^2}, \quad (j = 1, 2, 3, 4) \tag{8}$$

- f_{Rj} [MPa] is the residual flexural tensile strength corresponding to $CMOD_j$;
- b [mm] is the specimen width;
- l [mm] is the span length;
- F_j [N] is the load corresponding to $CMOD_j$, see figure 8;
- h_{sp} [mm] is the distance between the notch tip and the top of the specimen (125 mm);

The limit of proportionality f_L, as defined in EN 14651 [3], can be determined by applying the following equation:

$$f_L = \frac{3F_L \cdot l}{2b \cdot h_{sp}^2}. \tag{9}$$

where the load value F_L shall be determined by drawing a line at a distance of 0,05 mm and parallel to the load axis of the load-CMOD or load-deflection diagram and taking as FL the highest load value in the interval of 0,05 mm, see figure 7 (§ 9.2 [3]) .

7 Test set-up for behaviour in tension § 5.6, page 145 [1]

With regard to the behaviour in tension, which is the most important aspect of FRC, various test methods are possible. Bending tests can be carried out aiming at determining the load- deflection relation. Nominal values of the material properties can be determined by performing a three-point bending test on a notched beam according to EN 14651 (figure 8 (a)). The diagram of the applied force (F) versus the deformation must be produced (figure 8 (b)). The deformation is generally expressed in terms of crack mouth opening displacement (CMOD), which is the opening of the notch at the bottom face of the beam (figure 8).

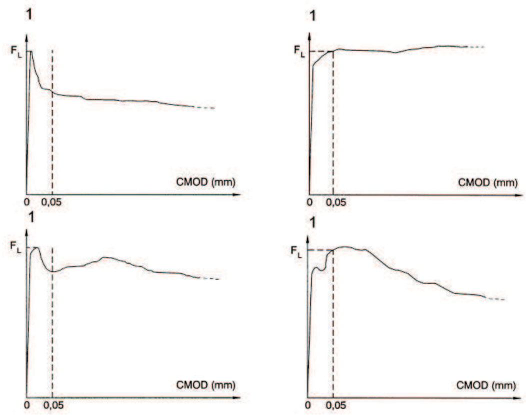

Figure 7: Load-CMOD diagrams and FL.

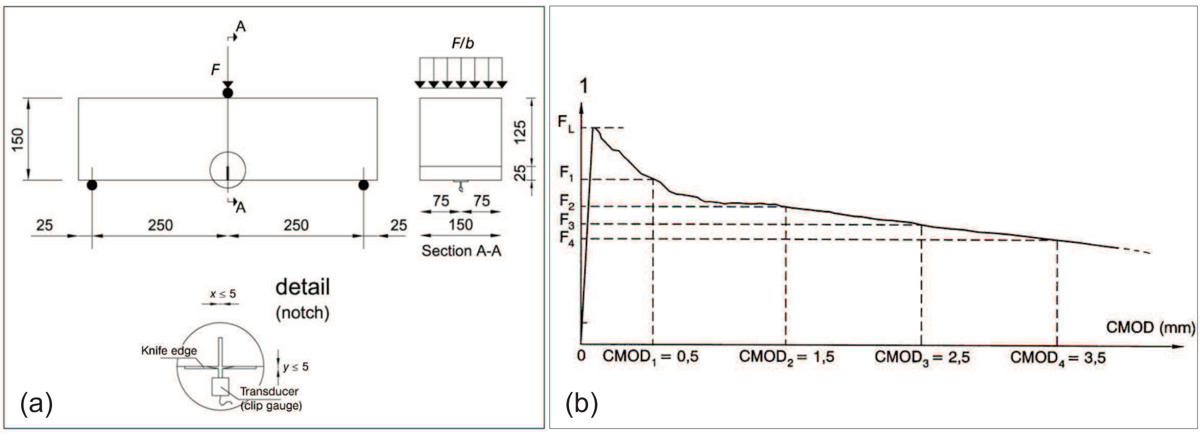

Figure 8: Test set-up required by EN 14651 (dimensions in [mm]) (left). Typical load FCMOD curve for plain concrete and FRC (right).

(??? non capisco cosa vuoldire questo limite)Note that, fiber reinforcement can substitute (also partially) conventional reinforcement at ultimate limit state, if the following relationships are fulfilled, see equations 5.6-2 and 5.6-3 of [1]:

$$\frac{f_{R1k}}{f_{Lk}} > 0.4, \quad \frac{f_{R3k}}{f_{Lk}} > 0.5. \tag{10}$$

References

[1] FIB - Federation Internationale du Beton, 2010, fib Model Code for Concrete Structures.

[2] CNR - Consiglio Nazionale delle Ricerche, CNR-DT 204, 2006, Istruzioni per la Progettazione, lEsecuzione ed il Controllo di Strutture di Calcestruzzo Fibrorinforzato.

[3] EN - EUROPEAN STANDARD, EN 14651, 2005, Test method for metallic fibered concrete - Measuring the flexural tensile strength (limit of proportionality (LOP), residual).