Tecnica delle costruzioni meccaniche – Scritto del 4 luglio 2016

Esercizio n° 1 (15 punti)

Si consideri l'albero di verricello riportato in figura: ad un'estremità è montato il tamburo con la fune a cui viene agganciato il peso da sollevare, all'altra estremità la ruota dentata a denti elicoidali che comanda il moto. L'albero è montato su due cuscinetti: quello vicino alla ruota agisce come cerniera e l'altro come carrello. Le forze esterne agenti sul sistema sono indicate e sono rispettivamente Ft=forza tangenziale Fr= forza radiale Fa=forza assiale alla ruota, W=peso da sollevare.

Si deve condurre la verifica di resistenza a snervamento dell'albero. In particolare si richiede:

1) Determinare le reazioni vincolari e disegnare il diagramma di corpo libero dell'albero
2) Rappresentare i diagrammi quotati di azione interna, riportando le convenzioni di segno adottate
3) Condurre la verifica a snervamento secondo il criterio di Von Mises e riportare il valore del coefficiente di sicurezza
4) Rappresentare lo stato di tensione sul volume elementare superficiale, nel punto critico dell'albero soggetto alla tensione di Navier massima positiva
5) Con riferimento al suddetto punto rappresentare i cerchi di Mohr delle tensioni e determinare le tensioni principali

Dati:

L_1= 200 mm L_2= 1000 mm L_3= 300 mm

R_1= 180 mm R_2= 150 mm

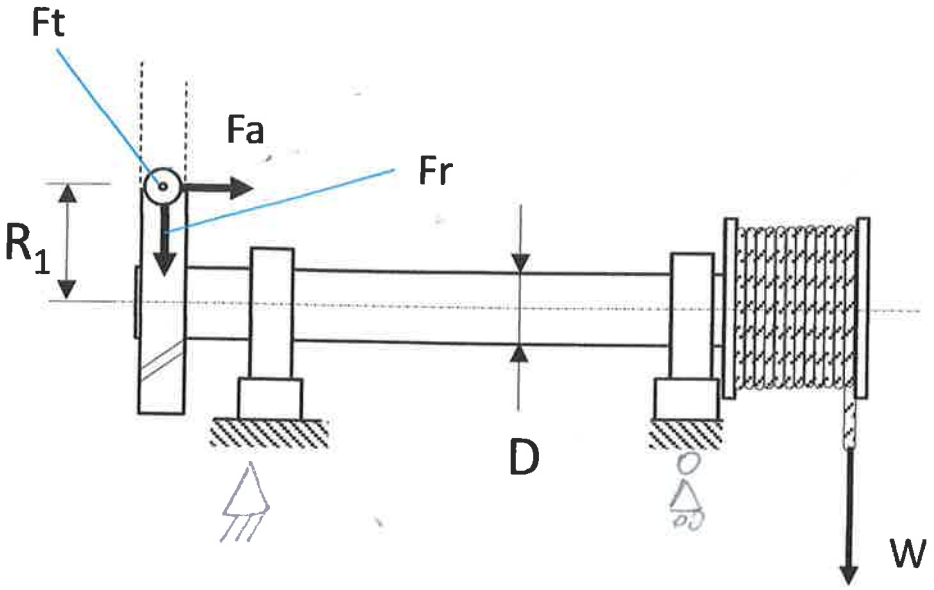
D= 50 mm

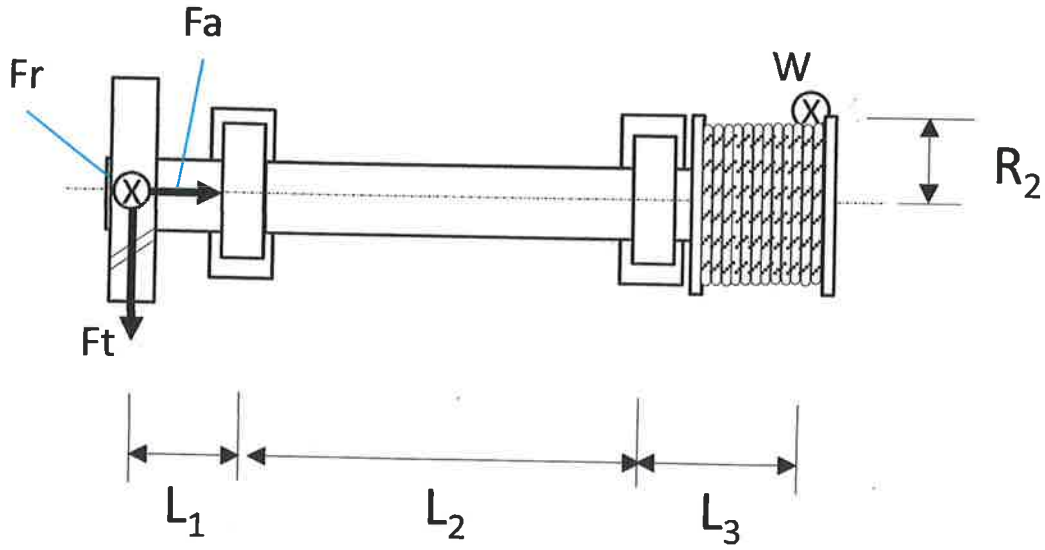
W=10000 N = 10 KN ~ 20 passo

Fr=0.377Ft = 3244,54 N

Fa=0.268Ft = 2233,24 N

Materiale: acciaio C40 bonificato: E= 210 GPa, σ_{ys}= 475 MPa


Ft
Fa
Fr
R1
D
W


Fa
Fr
W
R2
Ft
L1
L2
L3

Esercizio n° 2 (17 punti)

La trave riportata in figura funge da binario per una slitta che può muoversi lungo la via di corsa indicata ($0<z_1<L$) Ad un'estremità la trave è incastrata, mentre all'altra estremità è supportata da una barra verticale di sezione tubolare, vincolata alla trave ed al telaio mediante cerniere.

Si devono realizzare le verifiche di resistenza necessarie per i due elementi strutturali. In particolare si richiede

- Calcolare le reazioni vincolari e disegnare il diagramma di corpo libero per una posizione generica z1 della slitta
- Verificare che la deformabilità assiale della barra tubolare ha incidenza trascurabile sui valori di reazione vincolare
- Discutere le verifiche strutturali necessarie per i due elementi ed identificare le posizioni critiche della slitta per le suddette verifiche
- In corrispondenza delle posizioni critiche riportare i diagrammi quotati di azione interna
- Disegnare una deformata plausibile della struttura

Dati:

L= 2000 mm S= 800 mm

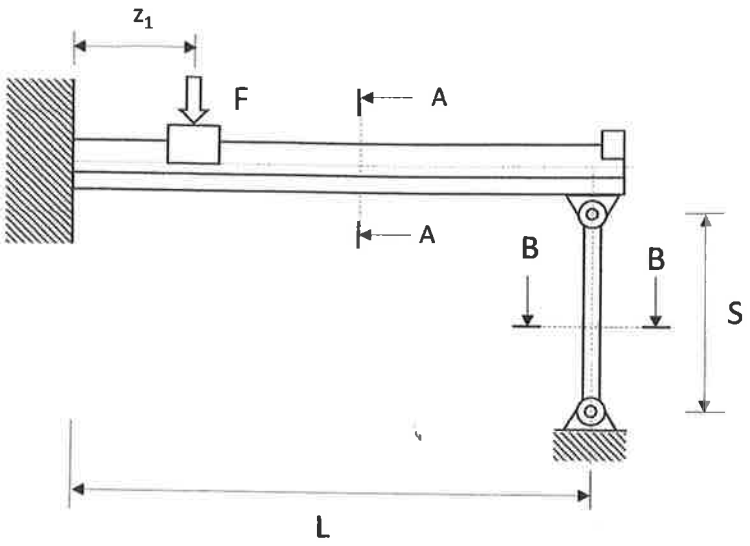
W_1= 100 mm W_2= 80 mm H_1= 120 mm H_2= 100 mm

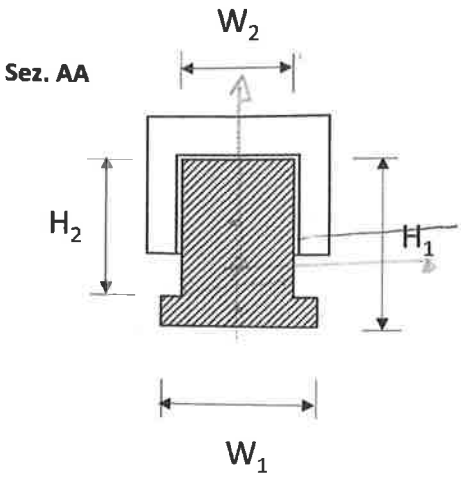
D= 50 mm s=4 mm

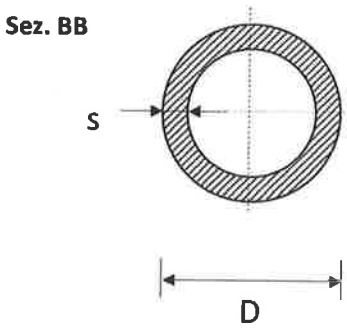
F= 15000 N

Materiale: Acciaio S235: E=210 GPa; σ_{ys}= 235 MPa

Quesito supplementare: verificare se la freccia massima si mantiene entro 0.5 mm


z_1
F
A
A
B
B
S
L

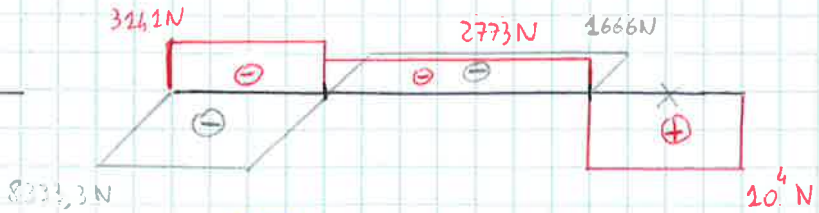

Sez. AA
W_2
H_2
H_1
W_1


Sez. BB
s
D

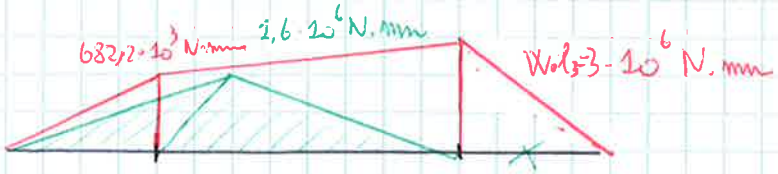
diagrammi:

N

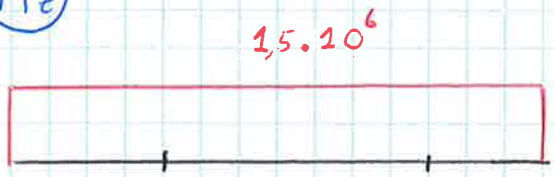
T



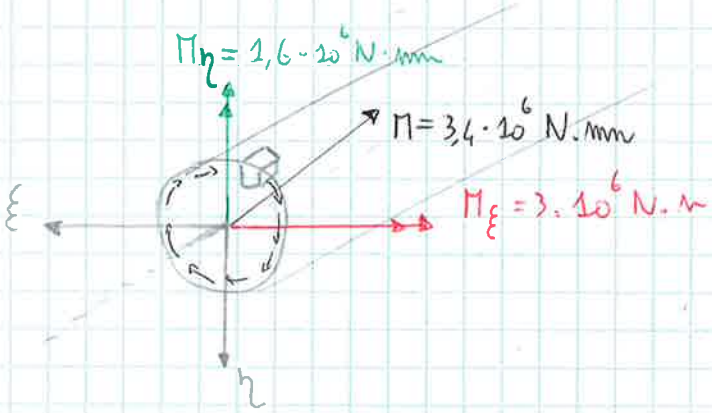
M_F

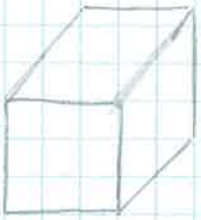


M_z



Verifica di sherramento





Vedendo

$$\sigma^+ = \frac{M \cdot R}{I_\xi} = \frac{\Pi \cdot R}{\frac{\pi \cdot R^4}{4}} = 277 \text{ MPa}$$

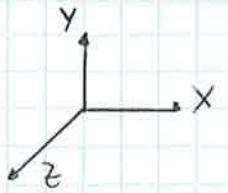
$$\tau = \frac{M_z \cdot R}{I_\rho} = \frac{1{,}5 \cdot 10^6 \cdot 25}{\frac{\pi \cdot R^4}{4}} = 122{,}23 \text{ MPa}$$

$$\sigma_s = \sqrt{277^2 + 3 \cdot 122^2} = 348 \text{ MPa} \quad < 475$$

$$f = \frac{475}{348} = 1{,}36$$

PROVA, Flessione deviata

schema equivalente:



analisi cinematica:

$gdV = 5 \qquad gdI = 6$

Momento torsionale libero, ma AUTOEQUILIBRATO:

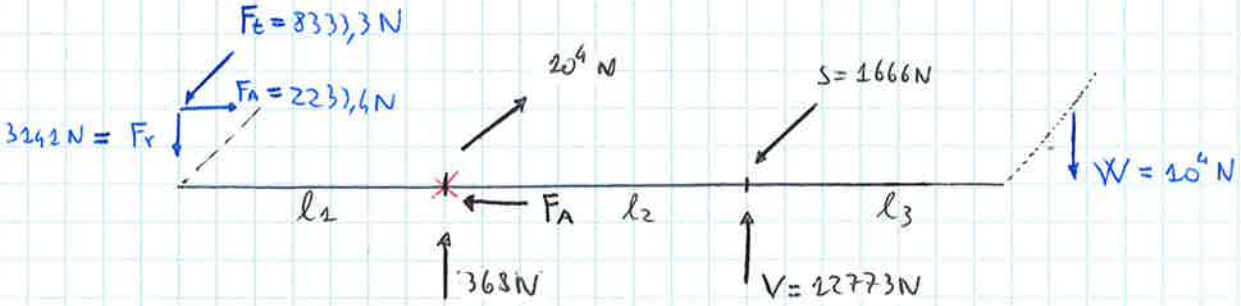
$$F_t \cdot R_1 = W \cdot R_2 \quad \dashrightarrow \quad F_t = \frac{W \cdot R_2}{R_1} = 8333{,}3\ N$$

$F_R = 0{,}377 \cdot 8333 = 3141{,}54\ N$

$F_A = 0{,}268 \cdot 8333{,}3 = 2233{,}32\ N$

$W = 10^4\ N$

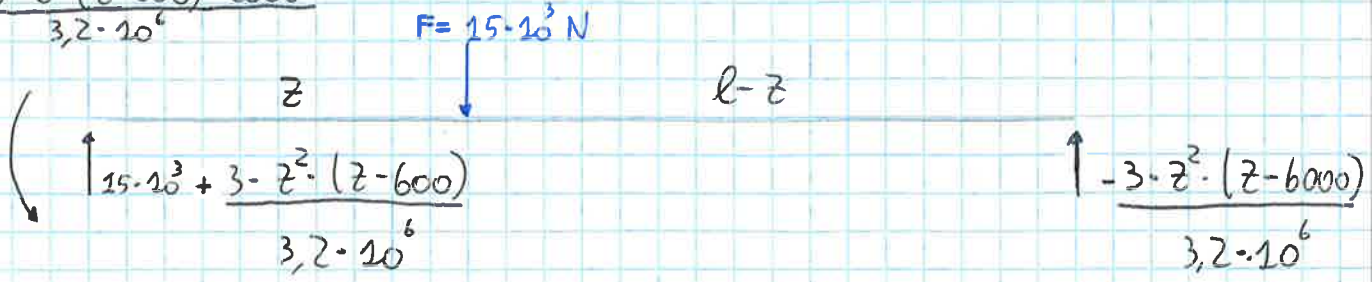
calcolo delle reazioni:



$$F_r \cdot l_1 - W \cdot (l_2 + l_3) + V \cdot (l_2) - F_A \cdot R_1 = 0 \quad \dashrightarrow \quad V = \frac{-F_r \cdot l_1 + W \cdot (l_2 + l_3) + F_A \cdot R_1}{l_2} = 12773$$

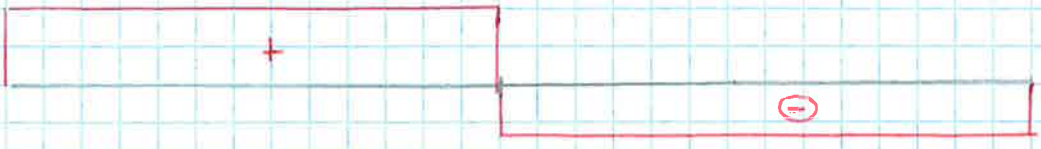
$$F_t \cdot l_1 = S \cdot l_2 \quad \dashrightarrow \quad S = \frac{F_t \cdot l_1}{l_2} = 1666{,}6\ N$$

$15 \cdot 10^3 \cdot z + \frac{3 \cdot z^2 \cdot (z-600) \cdot 2000}{3,2 \cdot 10^6}$

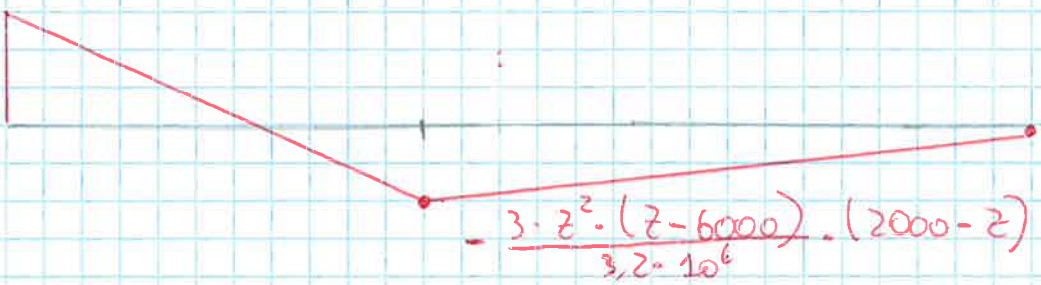


diagrammi generici:

(T)



(Π_F)

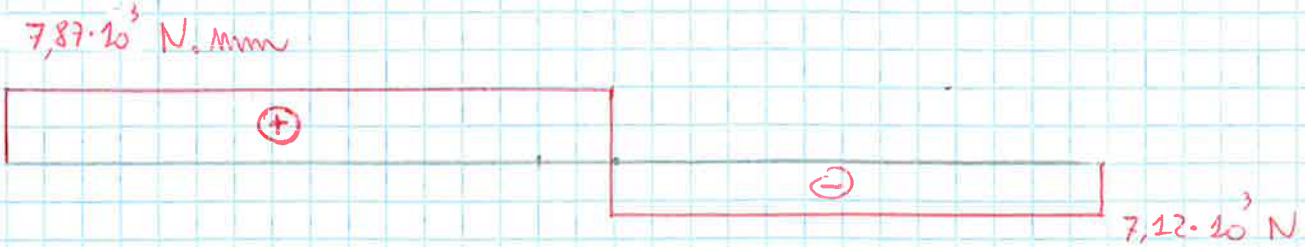


calcolo z critico:

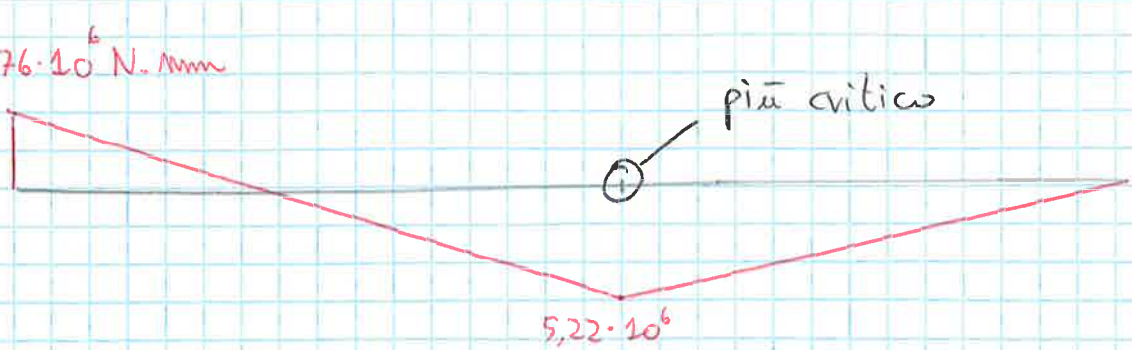
$\frac{d\Pi_z}{dz} = 0 \longrightarrow z = 126795 \text{ mm}$

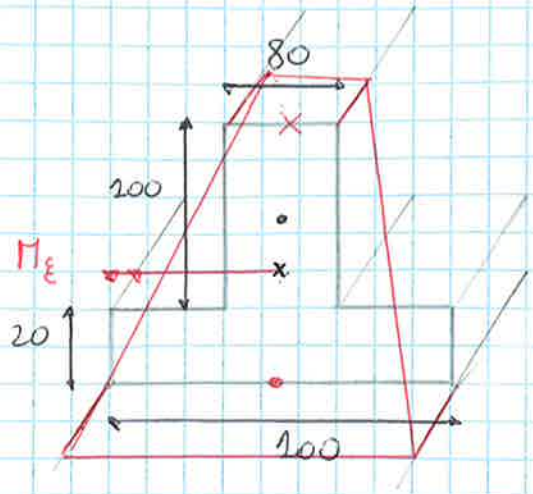
diagrammi z critico:

(T)



(M)





$$y_G = \frac{\delta X}{A_{TOT}} = \frac{(100\cdot 80)\cdot(70) + (100\cdot 20)\cdot 10}{100\cdot 80 + 20\cdot 100} = 58$$

$$I_\xi = \frac{80\cdot 100^3}{12} + (80\cdot 100)\cdot(70-58)^2 + \frac{100\cdot 20^3}{12} + (100\cdot 20)\cdot(58-10)^2$$

$$\doteq 1{,}24\cdot 10^7$$

$$\sigma = \frac{M_\xi \cdot h}{I_\xi} = \frac{5{,}22\cdot 10^6\cdot(120-58)}{1{,}24\cdot 10^7} = 26{,}1\ MPa$$

Dovrò: $N = 7{,}12\cdot 10^3\ N$

verifica deformabilità:

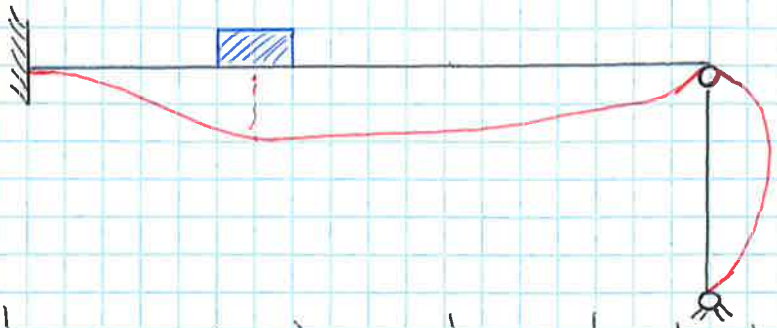
$$\delta = \frac{X}{K} \qquad K = \frac{E \cdot A}{L} = 251,738 \cdot 10^3 \frac{N}{mm}$$

$$\delta = \frac{7,12 \cdot 10^3}{251 \cdot 10^3} = 0,046$$

Esame 4 luglio 2016:

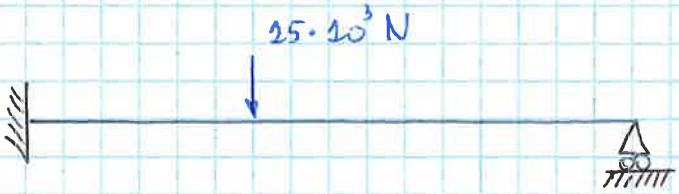
schema equivalente:

6 gdl → 7 gdV → IPER

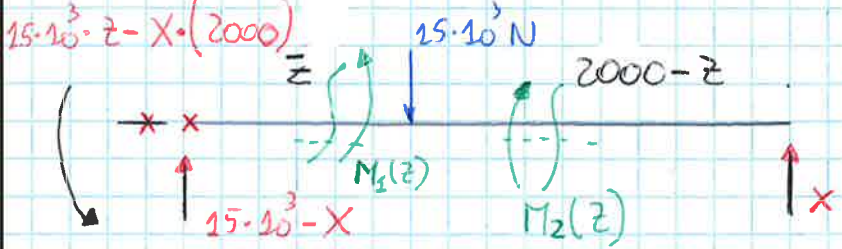


Calcolo reazioni applicando il teorema di menabrea:

(1) CONFIGURAZIONE ISO associata



Reazioni:

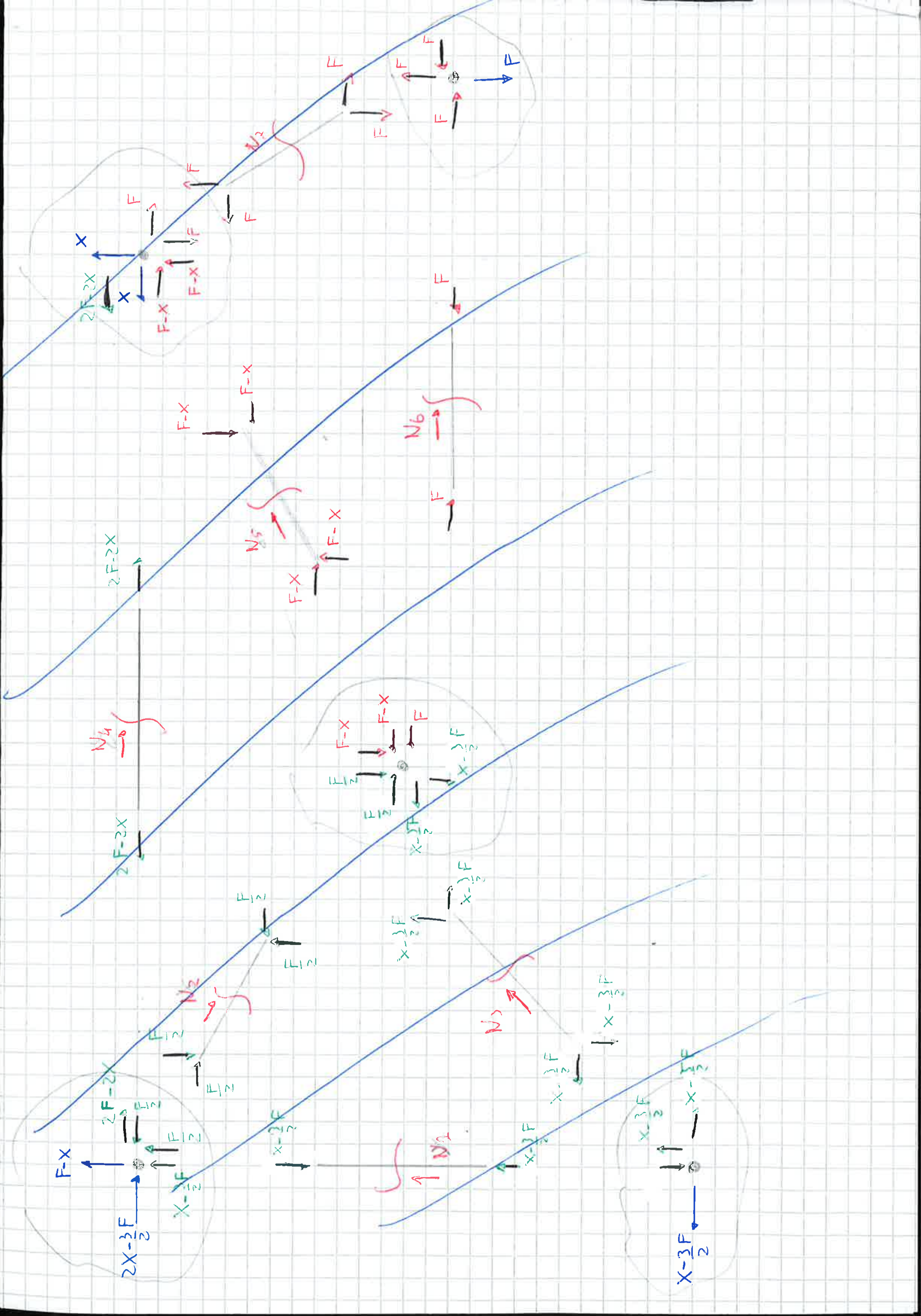


$$M_1(z) = (15 \cdot 10^3 - X) \cdot t - [15 \cdot 10^3 \cdot z - X \cdot (2000.)] \qquad 0 < t < z$$

$$M_2(z) = X \cdot t \qquad 0 < t < 2000 - z$$

$$\mathcal{E} = \frac{1}{2EI} \int_0^z M_1^2 \, dt + \frac{1}{2EI} \int_0^{2000-z} M_2^2 \, dt$$

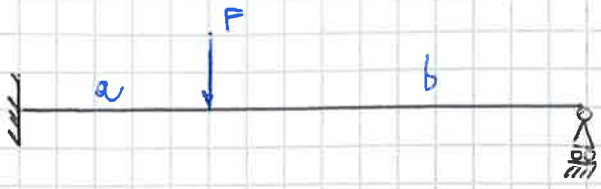
$$\frac{d\mathcal{E}}{dX} = 0 \quad \dots \quad X = \frac{-3 \cdot z^2 \cdot (z - 600)}{3{,}2 \cdot 10^6}$$


F
F-X
2F-2X
X
N_1
N_2
N_3
N_4
N_5
N_6
N_7
$\frac{F}{2}$
$X-\frac{3F}{2}$
$2X-\frac{3F}{2}$

ESERCIZIO GUIDA

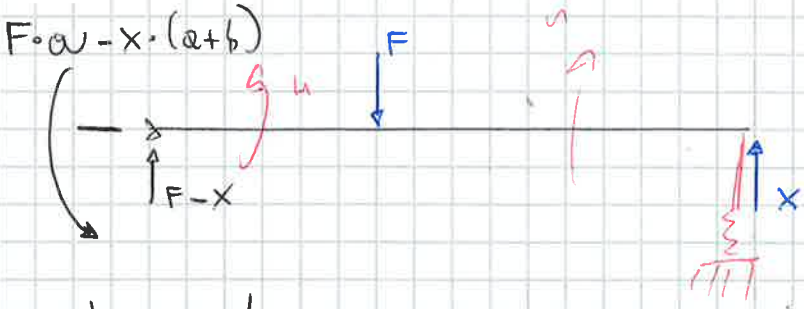
con Formulario

Schema equivalente:



analisi cinematica: IPER x 1 volta

Config **isostatica**:

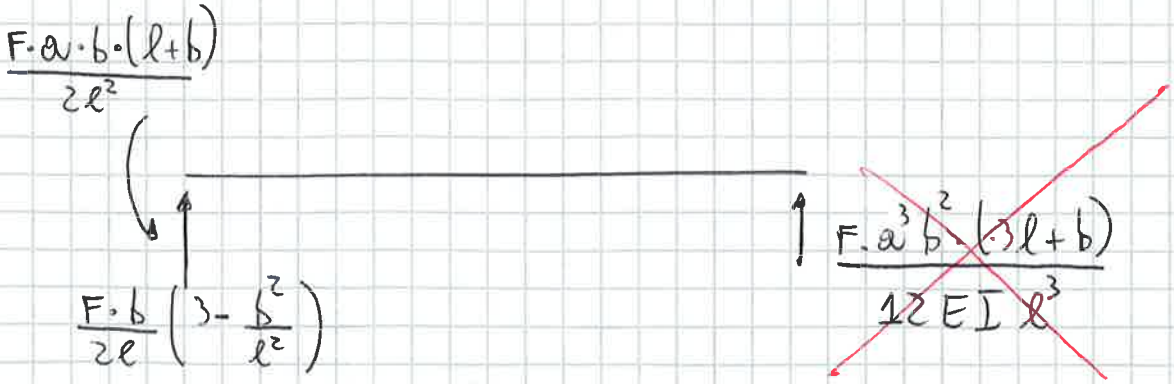


applico teo. MENABREA:

$$\xi = \frac{1}{2EI}\int_0^a N_1^2\,dz + \frac{1}{2EI}\int_0^b N_2^2\,dz$$

$$\frac{d\xi}{dx} = 0 \quad \Longrightarrow \quad X = \frac{F\cdot a^2\cdot(2l+b)}{2l^3}$$

diagramma corpo libero:



diagrammi:

(T)

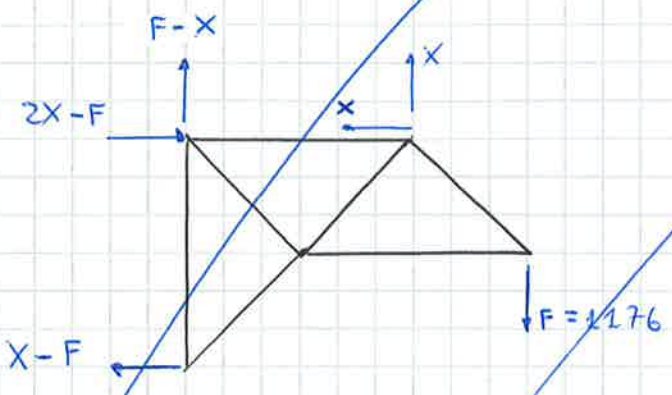
(N)

determino punto più critico:

$$\frac{d\Pi(z)}{da} = \frac{d}{da}\left(\frac{F \cdot a^2 \cdot (l-a)(l-a+2l)}{2l^3}\right)$$

$$\Downarrow$$

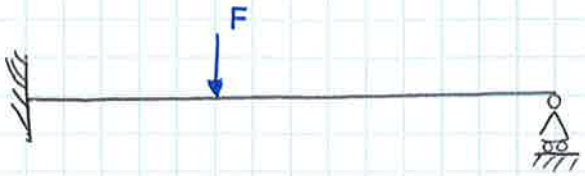
$$\boxed{a = 0{,}6\,l}$$



X = 1357,33

$\delta = -0{,}000488$ 2,33 mm

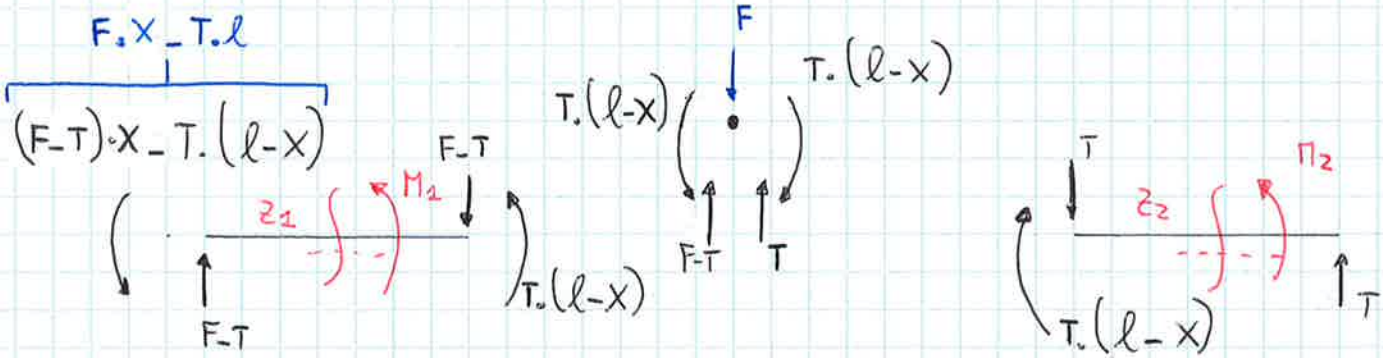
esame 4 luglio 2016: (ME ; the official)



1 volta IPER

Reazioni:

Spacco:

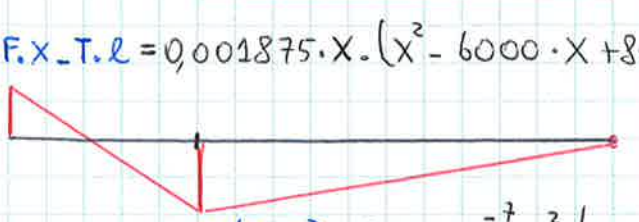


diagrammi:

(N)

(T)

(M)



$M_1(z_1) = (F\_T)\cdot z_1 - [F.X - T.\ell]$ $\qquad 0 < z_1 < X$

$M_2(z_2) = T.(\ell - X) - T\cdot z_2$ $\qquad 0 < z_2 < \ell - X$

$$y_1'' = \frac{M_1}{E \cdot I} = \frac{(F-T)\cdot z_1 - [F \cdot X - T \cdot \ell]}{E \cdot I}$$

$$y_1' = \frac{(15000-T)\cdot z_1^2/2 - [15000 \cdot X - T \cdot 2000]\cdot z_1 + C_1}{2{,}75 \cdot 10^{16}}$$

$$y_1 = \frac{(\overset{15000}{F} - T)\cdot z_1^3/6 - [\overset{15000}{F} \cdot X - T \cdot \overset{2000}{\ell}]\cdot z_1^2/2 + C_1 z_1 + C_2}{2{,}75 \cdot 10^{26}}$$

$$y_2'' = \frac{T\cdot(2000-x) - T \cdot z_2}{2{,}75 \cdot 10^{16}}$$

$$y_2' = \frac{T\cdot(2000-x)\cdot z_2 - T \cdot z_2^2/2 + C_3}{2{,}75 \cdot 10^{16}}$$

$$y_2 = \frac{T\cdot(2000-x)\cdot z_2^2/2 - T \cdot z_2^3/6 + C_3 z + C_4}{2{,}75 \cdot 10^{16}}$$

condizioni al contorno:

$$y_1(z_1=0) = 0$$

$$y_1'(z=0) = 0$$

$$y_2(z_2 = 2000-x) = 0$$

$$y_1(z_1=x) = y_2(z_2=0)$$

$$y_1'(z_1(x) = y_2'(z_2=0)$$

$t = -3{,}375 \cdot 10^{-7} x^2 \cdot (x - 6000)$

$C_3 = 4{,}68 \cdot 10^{-7} x^2 (x^3 - 10^4 x^2 + 2{,}4 \cdot 10^7 x - 1{,}6 \cdot 10^{10})$

$C_4 = 1{,}56 \cdot 10^{-7} x^3 \cdot (x^3 - 12000 \cdot x^2 + 3{,}6 \cdot 10^7 \cdot x - 3{,}2 \cdot 10^{10})$

$C_1 = 0$

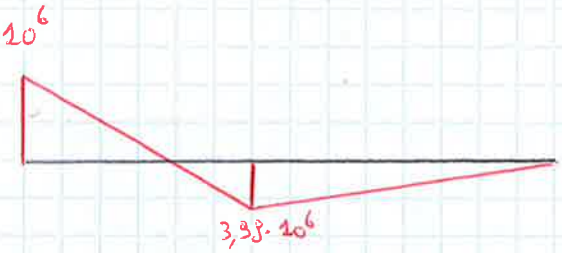
$C_2 = 0$

diagramma punto critico: X = 845 mm T = 3450N

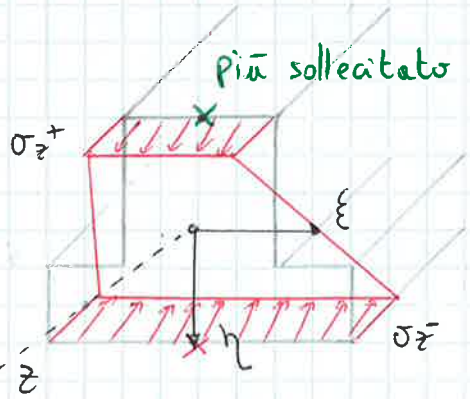
(N)

(T)

(M)



Verifiche sherramento:

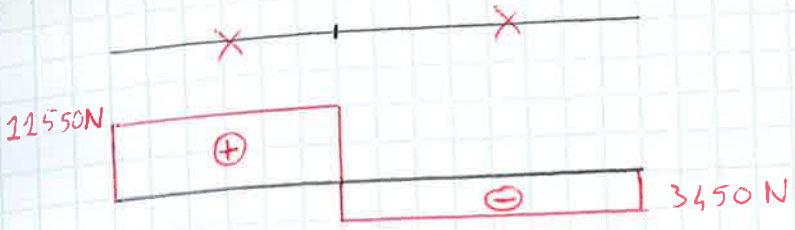


$\sigma_z = \frac{M \cdot \eta}{I} = \frac{5{,}77 \cdot 10^6 \cdot 62\,mm}{1{,}31 \cdot 10^7\,mm^4} = 26{,}42\,MPa$

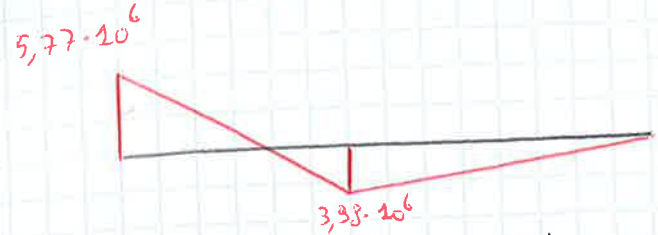
VON MISES: $\sigma_{eq} = \sqrt{\sigma_z^2 + 3\tau^2} = 26{,}42\,MPa < 235\,MPa$

(N)

(T)

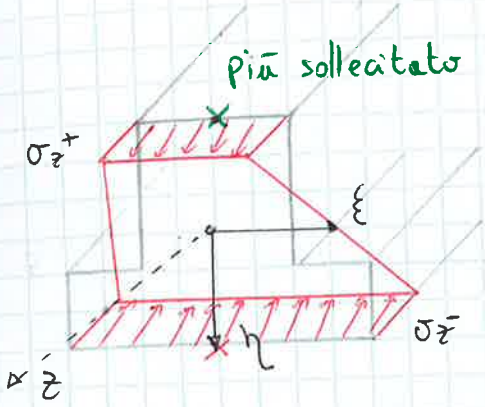


(M)



verifiche shervamento:

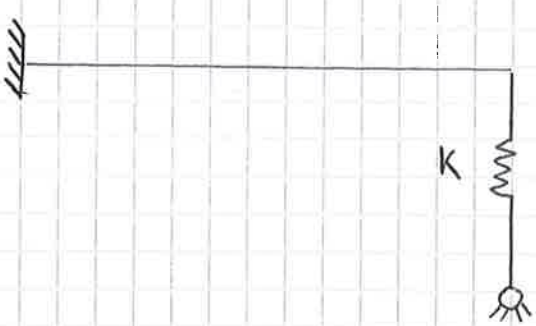
più sollecitato



$$\sigma_z = \frac{M \cdot \eta}{I} = \frac{5{,}77 \cdot 10^6 \cdot 62\,mm}{1{,}31 \cdot 10^7\,mm^4} = 26{,}42\,MPa$$

$$\sigma_{id} = \sqrt{\sigma_z^2 + 3\tau^2} = 26{,}42\,MPa < 235\,MPa$$

verifica de Formabilità assiale:

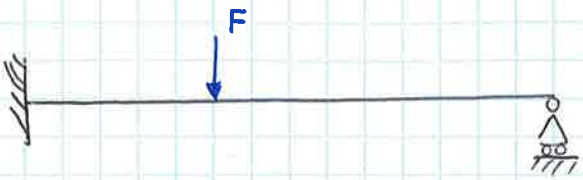


$$\delta = \frac{I}{K} \quad \text{con} \quad K = \frac{E \cdot A}{L} = \frac{E \cdot \left(\frac{\pi \cdot (D_e^2 - D_i^2)}{4}\right)}{L} = 251739\,\frac{N}{mm}$$

$$\delta = \frac{3450}{251739} = 0{,}0227\,mm \sim \text{trascurabile}$$

esame 4 luglio 2016: (ME ; the official)

1 volta IPER



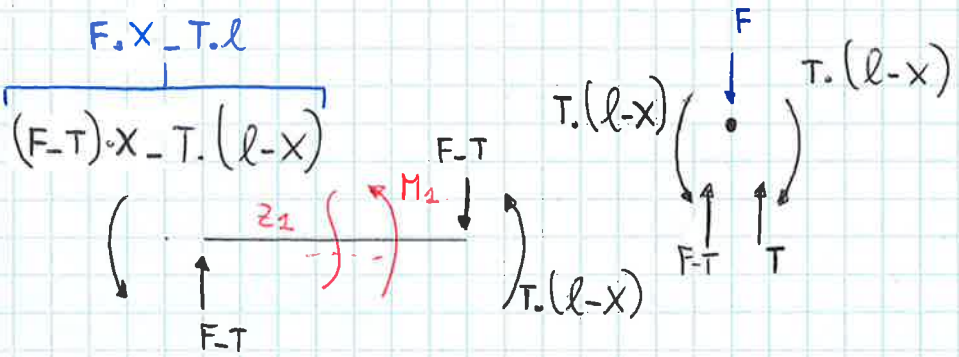
Reazioni:

$F.X - T.\ell$

Spacco:

$F.X - T.\ell$

$(F-T)\cdot X - T.(\ell-X)$



diagrammi:

(N)

(T)

$F_T =$

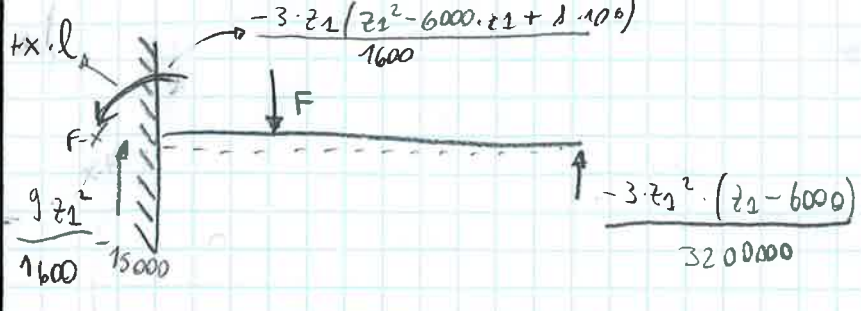
(M)

$F.X - T.\ell = 0{,}001875 \cdot X \cdot (X^2 - 6000 \cdot X + 8 \cdot 10^6)$

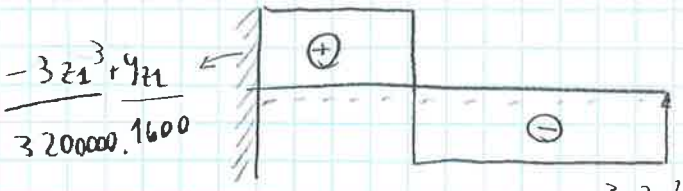
$T.(\ell - X) = 9{,}37 \cdot 10^{-7} \cdot X^2 \cdot (X - 6000)(X - 2000)$

$M_1(z_1) = (F-T).z_1 - [F.X - T.\ell]$ $\qquad 0 < z_1 < X$

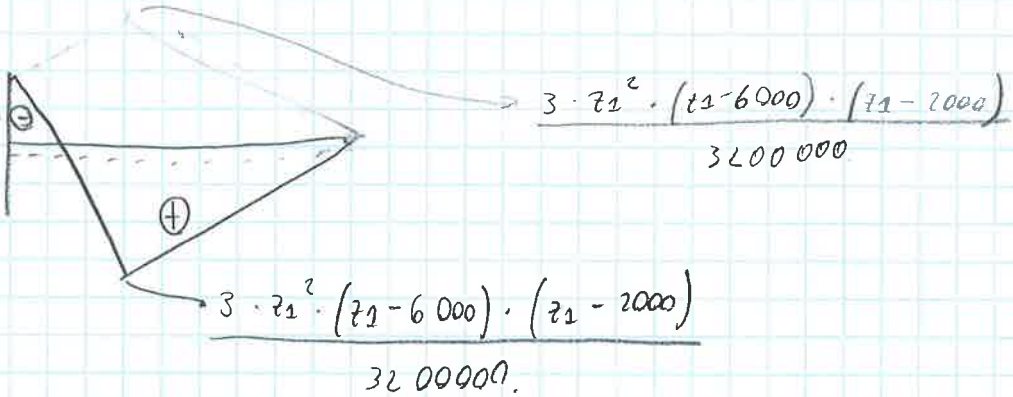
$M_2(z_2) = T.(\ell - X) - T.z_2$ $\qquad 0 < z_2 < \ell - X$



Ⓣ



Ⓜ

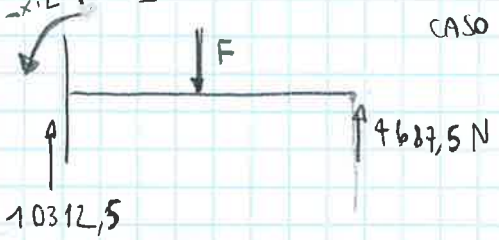


l' = L - t1 + t3

POSIZIONE CRITICA SLITTA È A METÀ DELLA TRAVE QUINDI $t_1 = 7500$ mm

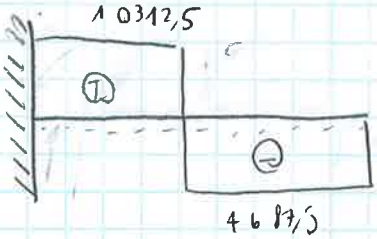
CASO IN CUI F È IN MEZZERIA

$-x \cdot L + F \cdot t_1 = 562500$

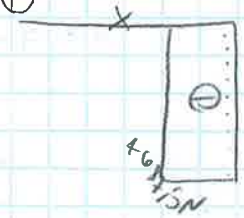


↳ $\frac{dM}{dz}$

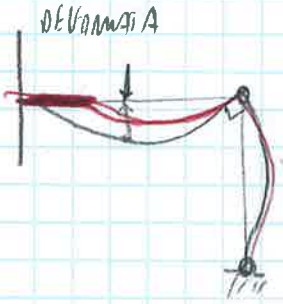
Ⓣ



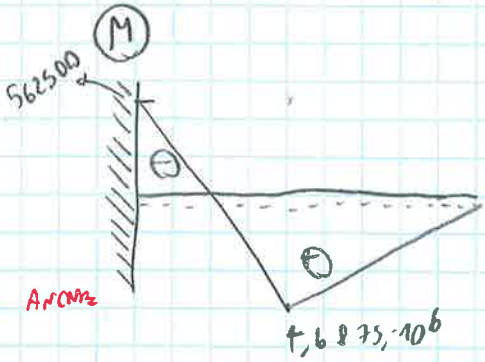
Ⓝ



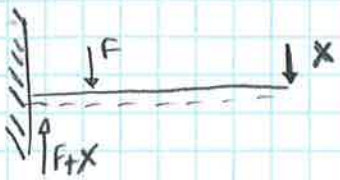
DEFORMATA



Ⓜ



+ FRECCIA MASSIMA
+ DEVO CONSIDERARE ANCHE
+ N DELL'ASTA?



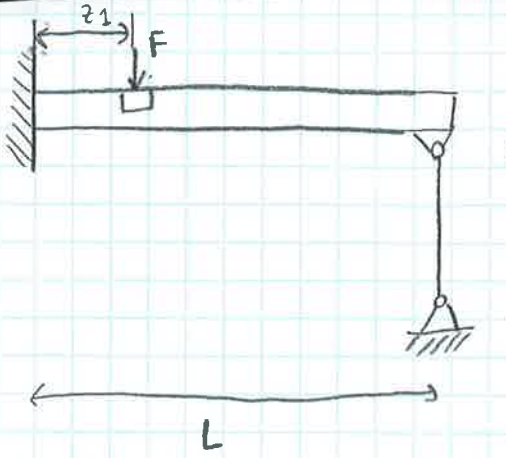
1° TRATTO

$M = -X \cdot z_2 \quad | \quad M' = -z_2$

2° TRATTO

$M = -X \cdot (L_1 - z_1 + z_3) - F \cdot z_3$

$M = (l - z_1 + z_3)$



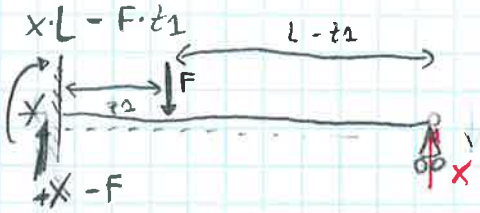
SLITTA SI MUOVE DA $0 < z_1 < L$

POSIZIONE GENERICA z_1 → CALCOLIAMO LE REAZ VINCOLARI

$F \cdot z_1$

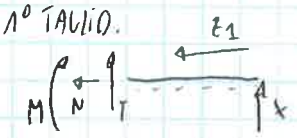
$+3 \cdot z_1^3$

3200000



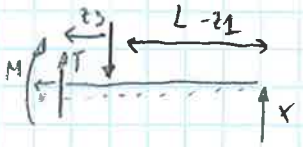
1 VOLTA IPERSTATICA

1° TAGLIO.



$M = +X \cdot z_2$ | $M' = z_2$

2° TRATTO



$M = +X \cdot (L - z_1 + z_3) - F \cdot z_3$ | M

$$\int_0^{L-z_1} (+X \cdot z_2)(z_2)\, dz_2 + \int_0^{z_1} (+X \cdot (L - z_1 + z_3) - F z_3) \cdot (L - z_1 + z_3) = 0$$

$$X = \frac{-3 z_1^2 (z_1 - 6000)}{32\,00000}$$

USO MENABREA E RISULTA CHE.

$$\int_0^{L-z_1} (+X \cdot z_1)^2 \, dz_2 + \int_0^{z_1} (+X (L - z_1 + z_3) - F \cdot z_3)^2 \, dz_3 = 0$$

$$X = \frac{-3 \cdot z_1^2 (z_1 - 6000)}{3200000}$$

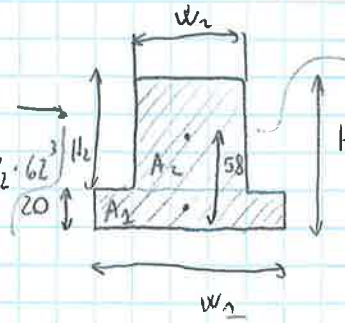
$W_1 = 100$ mm

$W_2 = 80$ mm

$H_1 = 110$ mm

$H_2 = 100$ mm

FRECCIA MASSIMA.



$$I = \frac{W_1 \cdot 58^3 - 20 \cdot (H_2 - 62)^3 + W_2 \cdot 62^3}{3} \simeq 1,25\ 10^7$$

MI DEVO RICAVARE IL BARICENTRO

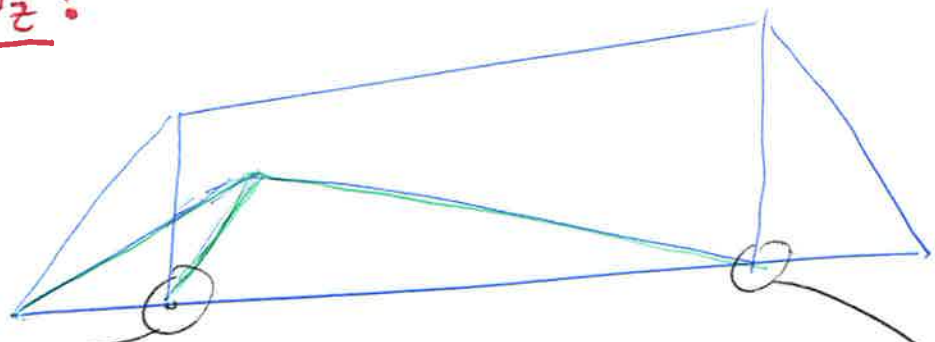
$$Y_G = \frac{10 \cdot (20\,100) + 70 \cdot (100 \cdot 80)}{(20 \cdot 100) + (100 \cdot 80)} = 58$$

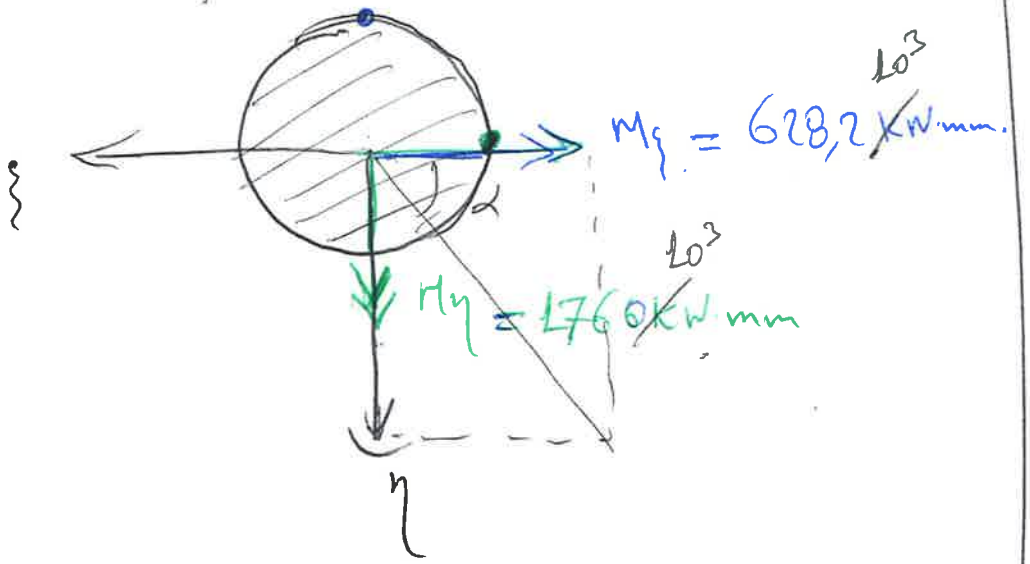
FRECCIA MASSIMA RISULTA CHE $A_{TOT} = 100\,00\ \text{mm}^2$

$$\frac{1}{2EI} \cdot \left[\int_0^{L/2} (46 + 8,5 + z_2)^2 + \int_0^{L/2} (+46 + 8,5 \cdot (L - 1000 + z_3) - F \cdot z_3)^2 \, dz_3 \right]$$

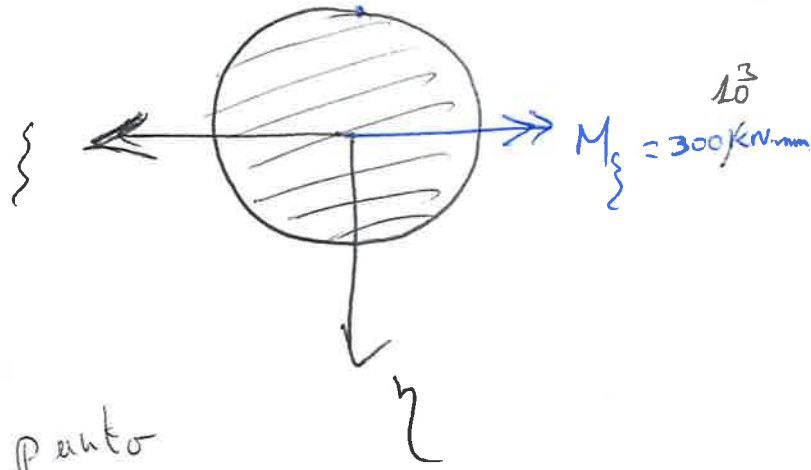
→ FRECCIA NON

Allegato per $\bar{\sigma}_z$:





$$\alpha = \operatorname{arctg}\left(\frac{M_\eta}{M_\xi}\right) = 79{,}43°$$



in questo punto
$\bar{\imath}$ trascurabile

eq. asse neutro

$$\sigma_z = -\frac{M_\xi}{I_\xi}\eta - \frac{M_\eta}{I_\eta}\xi - \frac{N}{A} = 0$$

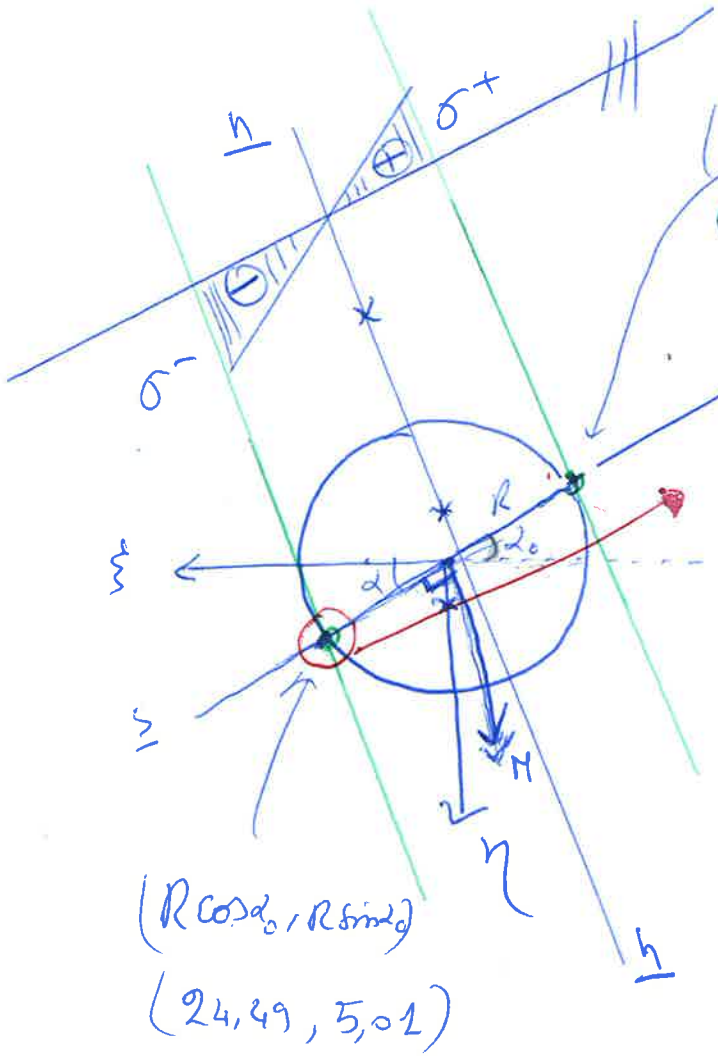
$$\eta = \left[-\frac{M_\eta}{I_\eta}\xi - \frac{N}{A}\right]\frac{I_\xi}{M_\xi}$$

$$= -\left(\frac{M_\eta}{M_\xi}\cdot\frac{I_\xi}{I_\eta}\right)\xi - \frac{N}{A}\cdot\frac{I_\xi}{M_\xi}$$

$$= -\frac{M_\eta}{M_\xi}\xi - \frac{N}{M_\xi}\frac{I_\xi}{A}$$

$$= -\left(\frac{1760\times10^3\,[N\cdot mm]}{628{,}2\times10^3\,[N\cdot mm]}\right)\xi - \frac{2933\,[N]\cdot\pi(25)^4/4}{628{,}2\times10^3\cdot\pi(25)^2} = -2{,}80\,\xi - 0{,}56$$

η	ξ
$-0,56$	0
$-3,36$	1

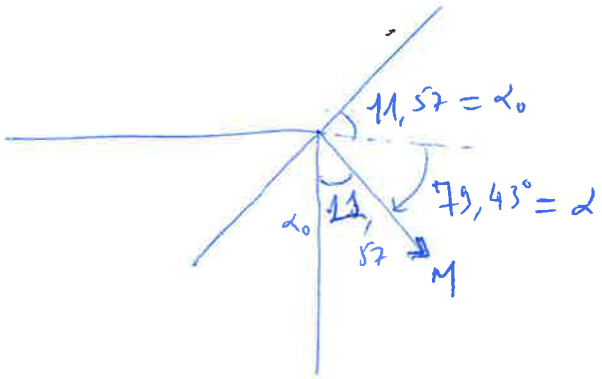


peggior caso è questo, xché

$$\sigma = -\frac{M_\xi \cdot \eta}{I_\xi} - \frac{M_\eta \cdot \xi}{I_\eta} - \frac{N}{A}$$

↓ xché η e $\xi = {}^{+}_{+}$ e sono tutti negativi!

X capire angolo α:



$$\sigma_z^{\textcircled{G}} = -\frac{628{,}2\times10^{3}}{\frac{\pi 25^4}{4}}\cdot(+5{,}01) - \frac{1760\times10^{3}}{\frac{\pi 25^4}{4}}\cdot(24{,}49) - \frac{2233{,}4}{\pi(25)^2}$$

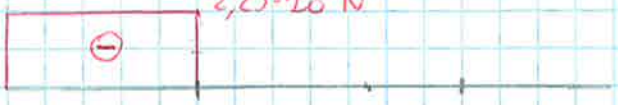
$$= -151\ \text{MPa}$$

$8{,}33 \cdot 10^3 \cdot 1200 - V \cdot 2000 = 0 \quad \therefore V = 9996\ N$

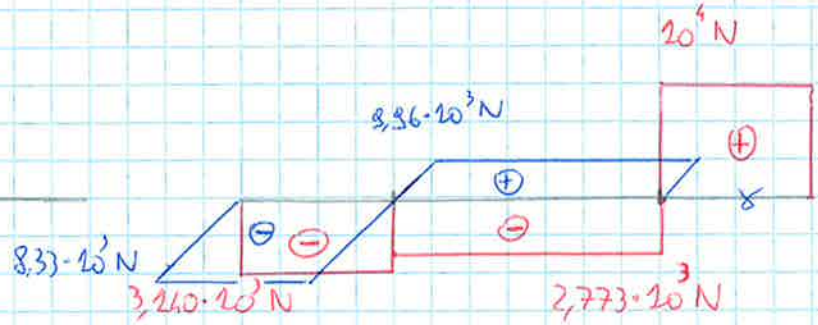
Verifico: $8{,}33 \cdot 10^3 \cdot 200 - 1{,}6 \cdot 10^3 \cdot 2000 \quad \dots \checkmark$

diagrammi:

(N)

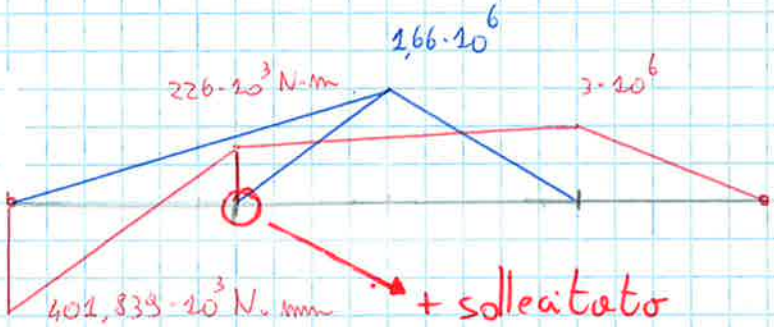


(T)



$10^4\ N$

Π_4

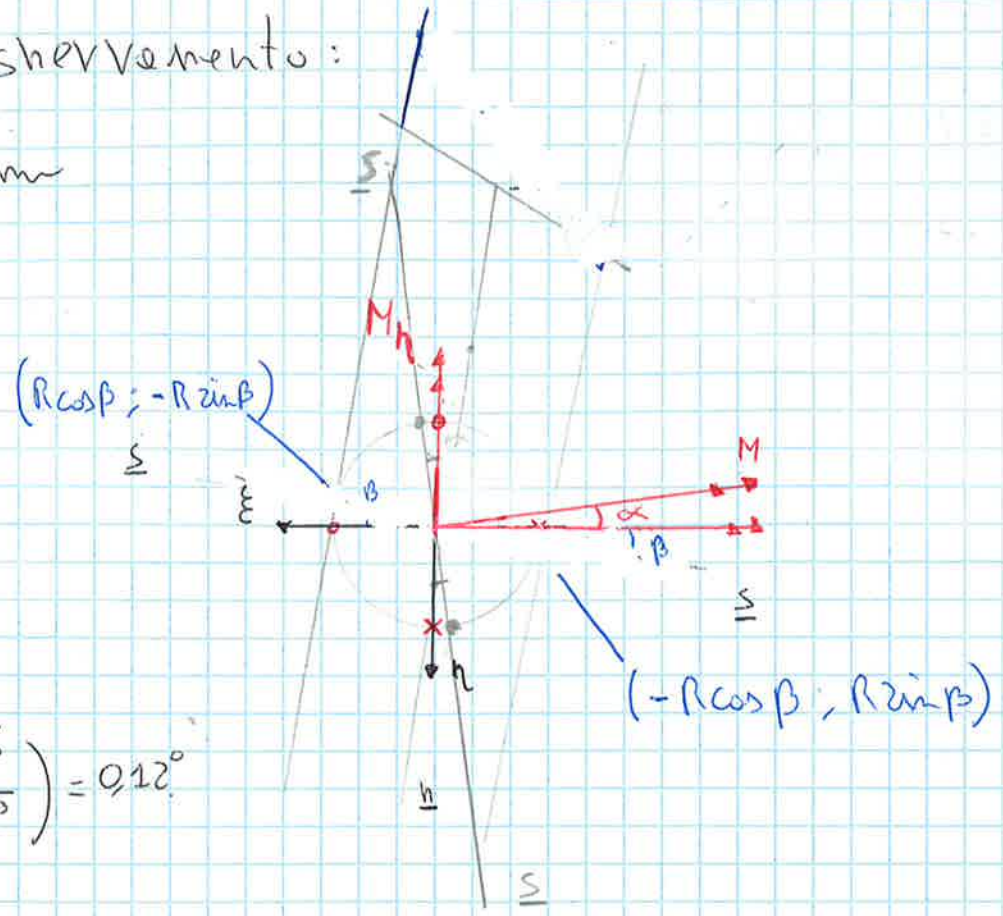


Π_2

verifiche di shervamento:

$M_\eta = 1{,}66 \cdot 10^6$ N·mm

$\Pi_\xi = 226 \cdot 10^3$ N·mm



$\alpha = tg\left(\frac{1{,}66 \cdot 10^6}{226 \cdot 10^3}\right) = 0{,}12°$

$\sigma_z = \frac{N}{A} \pm \frac{\Pi_\xi \cdot \eta}{I_\xi} \pm \frac{\Pi_\eta \cdot \xi}{I_\eta} = 0$

$I_\xi = I_\eta = \frac{\pi \cdot (D/2)^4}{4} = 3{,}06 \cdot 10^5$ mm^4

$\eta = -1{,}56 \quad X = 0$

$\eta = 5{,}85 \quad X = 1$

$\rightarrow \sigma_z = -1{,}13 - 0{,}72\,\eta + 5{,}42\,\xi = 0$

$\eta = \frac{5{,}42}{0{,}72}\,\xi - 1{,}13$ $\qquad \beta = \tan\left(\frac{5{,}42}{0{,}72}\right) = 0{,}13°$

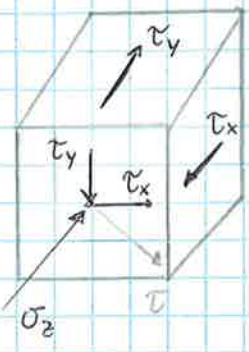
$\sigma_z^+\left(\overset{25}{R}\cos\beta ; -\overset{0}{R}\sin\beta\right) = 134$ MPa

$\sigma_z^- = (-R\cos\beta ; R\sin\beta) = 0$

$\tau = \frac{\Pi_z \cdot D/2}{I_p} = \frac{1{,}43 \cdot 10^6 \cdot 25}{6{,}12 \cdot 10^5} = 60{,}86$ MPa

$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2} = 170$ MPa

cubo elementare



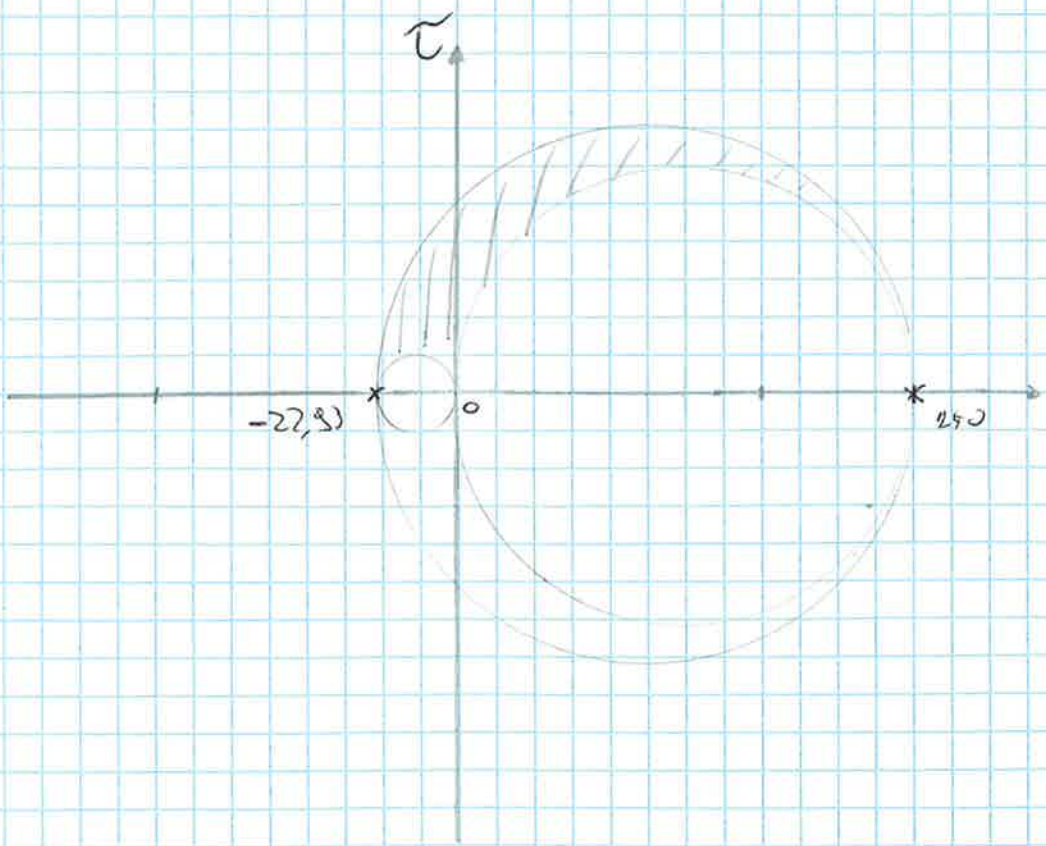
$\tau_{xz} = \tau \cdot \cos(90-\alpha) =$

$\tau_{yz} = \tau \sin(90-\alpha)$

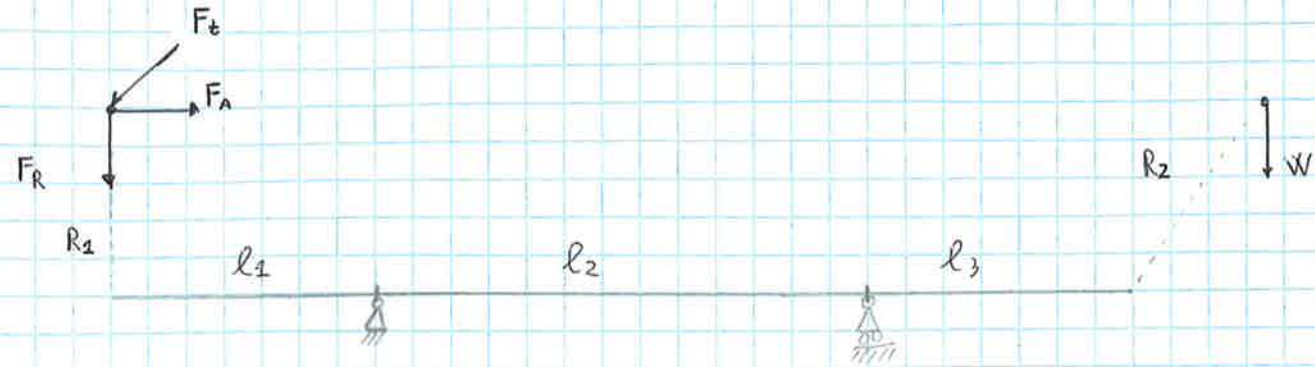
$$\sigma = \begin{bmatrix} 0 & 0 & 0,12 \\ 0 & 0 & 60 \\ 0,12 & 60 & 134 \end{bmatrix}$$

$\sigma = \det[\sigma - \lambda I] =$

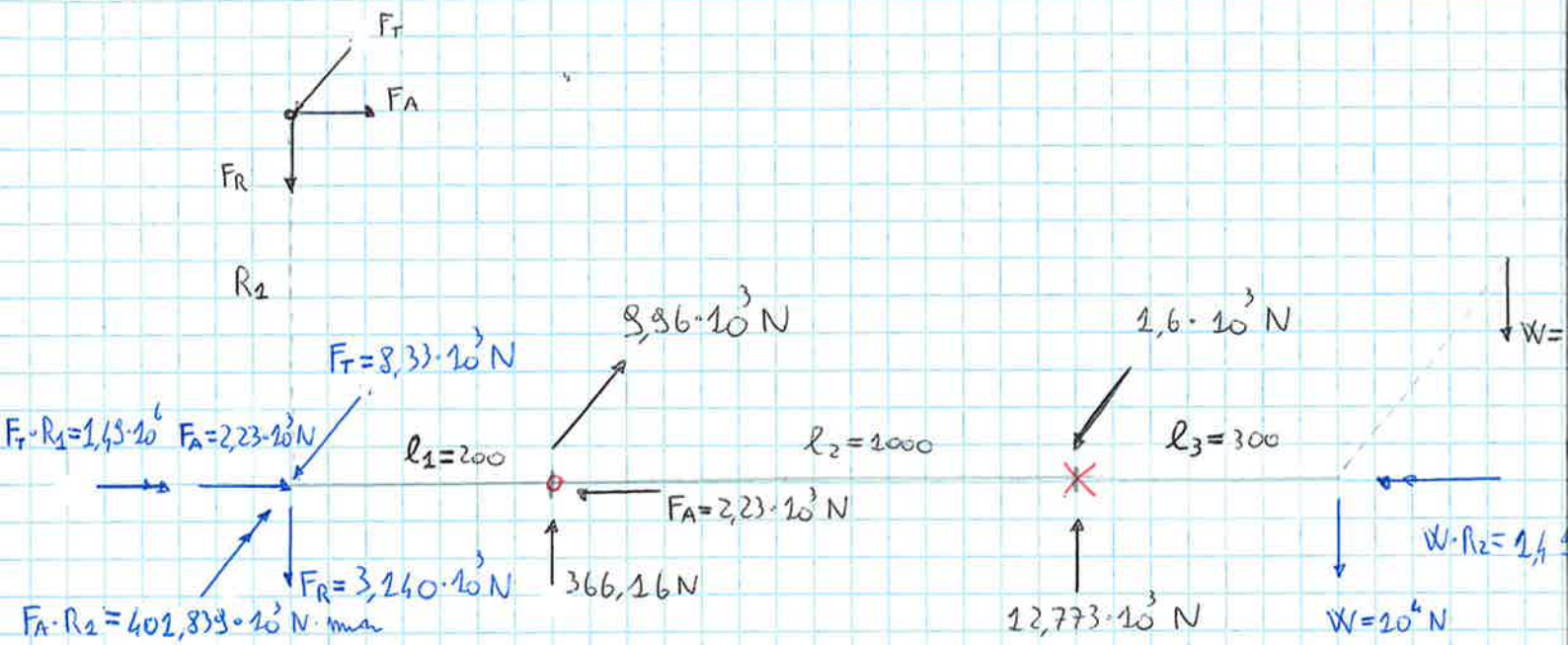
$X_I = 156,93 \quad X_{II} = -22,93 \quad X_{III} = 0$



Esame 4 luglio 2016: (FATTO 20 MAGGIO)



calcolo reazioni



(1) $F_T \cdot R_1 = W \cdot R_2$ ---- $F_T = \frac{W \cdot R_2}{R_1} = 8333,3\,N \rightarrow 8,33 \cdot 10^3\,N$

$F_R = 3140,41\,N$

$F_A = 2232,44\,N$

↺ $10^4 \cdot 300 + 401,839 \cdot 10^3 - 3,140 \cdot 10^3 \cdot 1200 - V \cdot 1000 = 0$

$V = -366,16\,N$

Verifico:

↻ $3,140 \cdot 10^3 \cdot 200 - 401,839 \cdot 10^3 + 12,77 \cdot 10^3 \cdot 1000 - 10^4 \cdot 1300 = 0$ ✓