① analisi cinematica: → ISO ; → IPER → n. volte IPER (esternamente / internamente) ; config. ISO ; MENABREA

MENABREA

$$\underset{\text{en. elastico deF.}}{\mathcal{E}} = \frac{1}{2EI_x}\int_0^{\ell} M^2\,dz + \frac{1}{2EA}\int_0^{\ell} N^2\,dz + \frac{1}{2}\frac{F_{molla}^2}{K} + \frac{1}{2}\cdot\frac{M_{molla}^2}{K_{ROT}} + \frac{1}{2GI_{p\,\text{polare}}}\int_0^{\ell} M_t^2\,dz$$

$K = \frac{E\cdot A}{L}$ $\qquad$ $G = \frac{E}{2(1+\nu)}$

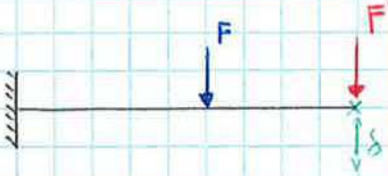
1 volta IPER: $\frac{d\mathcal{E}}{dX} = 0 \; \ldots\ldots \; X = \_\_$

2 volte IPER: $\begin{cases} \frac{d\mathcal{E}}{dX} = 0 \; \ldots\ldots \; X = \_\_ \\ \frac{d\mathcal{E}}{dY} = 0 \; \ldots\ldots \; Y = \_\_ \end{cases}$

SPOSTAMENTI:

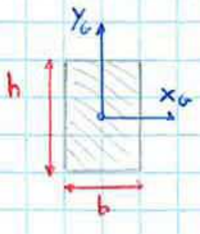
$\mathcal{E}$ = come prima .... $\delta = \frac{d\mathcal{E}}{dF}$

se:



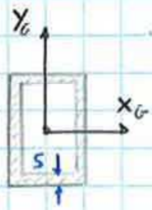
calcolo spostamento → inserisco F^* fittizia → $\mathcal{E} = \mathcal{E}(F; F^*)$ → $\frac{d\mathcal{E}}{dF^*} = \delta^*$ → $\lim_{F^*\to 0} \delta^* = \delta(F)$

MOMENTI INERZIA:



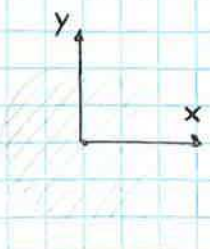
$I_{xG} = \frac{b\cdot h^3}{12}$

$I_{yG} = \frac{b^3\cdot h}{12}$



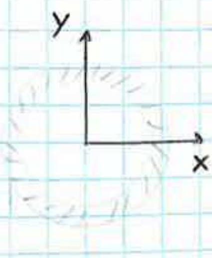
$I_{xG} = \frac{b\cdot h^3}{12} - \frac{(b-2S)(H-2S)^3}{12}$

$I_{yG} = \frac{b^3\cdot h}{12} - \frac{(b-2S)^3(H-2S)}{12}$



$I_x = I_y = \frac{\pi\cdot R^4}{4}$

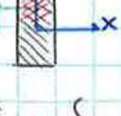
$I_p = 2I_x = 2I_y = \frac{\pi\cdot R^4}{2}$



$I_x = I_y = \frac{\pi}{4}\left[r_e^4 - r_i^4\right]$

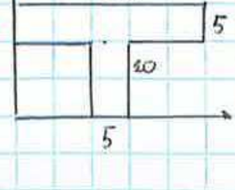
$I_p = \frac{\pi}{2}\left[r_e^4 - r_i^4\right]$

MOM. statico: $S = A\cdot d_y$



BARICENTRO: $X_G = \frac{S_y}{A_{tot}}$ $\qquad$ $Y_G = \frac{S_x}{A_{tot}}$

Esempio:

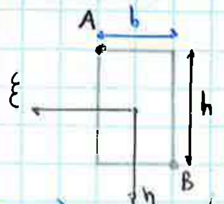


$X_G = \frac{S_y}{A_{tot}} = \frac{(5\cdot 10)\cdot 5 + (5\cdot 10)\cdot 5}{5\cdot 10\cdot 2}$

$Y_G = \frac{S_x}{A_{tot}} = \frac{(5\cdot 10\cdot 12{,}5) + (5\cdot 10)\cdot 5}{5\cdot 10\cdot 2}$

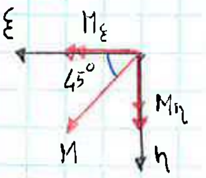
Verifiche snervamento:

$$\sigma = \pm \frac{N}{A} \pm \frac{M_\xi}{I_\xi}\cdot\eta \pm \frac{M_\eta \cdot \xi}{I_\eta}$$

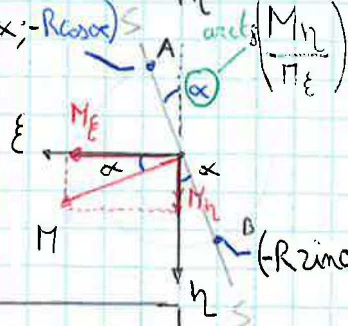


$$\sigma_{zA} = \pm \frac{M_\xi(-h/2)}{I_\xi} \pm \frac{M_\eta \cdot (b/2)}{I_\eta}$$

$$\sigma_{zB} = \pm \frac{M_\xi \cdot (-b/2)}{I_\xi} \pm \frac{M_\eta \cdot (h/2)}{I_\eta}$$



$$\overset{+}{\sigma} = \overset{-}{\sigma} = \frac{M \cdot R}{I_\xi}$$

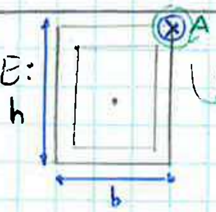


$(R\sin\alpha; -R\cos\alpha)$ $\arctan\left(\frac{M_\eta}{M_\xi}\right)$ $(-R\sin\alpha; R\cos\alpha)$

$$\sigma_{zA} = \pm \frac{M_\xi \cdot \overset{-R\cos\alpha}{\eta}}{I_\xi} \pm \frac{M_\eta \cdot \overset{R\sin\alpha}{\xi}}{I_\eta}$$

$$\sigma_{zB} = \pm \frac{M_\xi \cdot \overset{R\cos\alpha}{\eta}}{I_\xi} \pm \frac{M_\eta \cdot \overset{R\sin\alpha}{\xi}}{I_\eta}$$

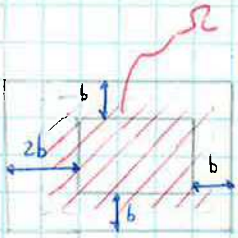
PRESSO-FLESSIONE:



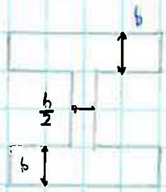
$$\sigma_{zA} = -\frac{N}{A} - \frac{M_\xi}{I_\xi}\cdot\frac{h}{2} - \frac{M_\eta}{I_\eta}\cdot b$$

τ generato da TORSIONE:

$$\tau = \frac{M_z \cdot R}{I_P}$$



$$\tau_{MAX} = \frac{M_T}{2\Omega\, b_{min}}$$



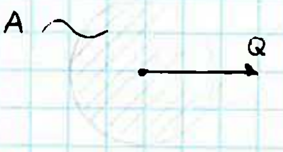
$$\tau_{MAX} = \frac{M_T}{I_T}\cdot b_{max}$$

$$I_T = \frac{\sum_i a_i \cdot b_i^3}{3}$$

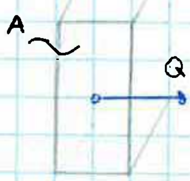
τ generato da taglio:

$$\tau_{MAX} = \frac{T \cdot S_{MAX}}{I \cdot b}$$

(sempre sul baricentro)



$$\tau_G = \frac{4}{3}\frac{Q}{A}$$



$$\tau_G = \frac{3}{2}\frac{Q}{A}$$

VON MISES: $\sigma_{eq_{\text{cartesiana}}} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - (\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z) + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)} \leq \frac{\sigma_{sy}}{\gamma_s}$

coeff. sicurezza

$$\sigma_{eq} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_I - \sigma_{III})^2}$$

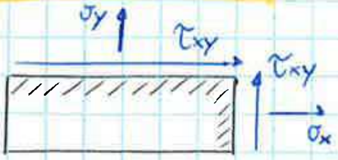
$$\boxed{\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2}}$$

$$\boxed{\gamma_s = \frac{\sigma_{sy}}{\sigma_{eq}}}$$

TRESCA: $\boxed{\sigma_{eq} = \sqrt{\sigma^2 + 4\tau^2}}$ $\sigma_{eq} = \max\left\{|\sigma_I - \sigma_{II}|; |\sigma_I - \sigma_{III}|; |\sigma_{II} - \sigma_{III}|\right\} \leq \frac{\sigma_{sy}}{\gamma_s}$

MOHR:

convenzione:



cambio segno

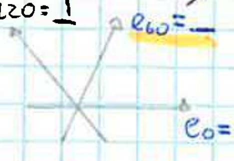
$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}$$

$$\sigma_{principali} = \det[\sigma - \lambda I] \rightarrow \sigma_I, \sigma_{II}$$

$$\theta_P = \frac{1}{2}\arctan\left[\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right]$$

graficamente: $P^I(\sigma_{xx}; \sigma_{xy})$ $P^{II}(\sigma_{yy}; \sigma_{xy})$

ROSETTA:



$e_{120} =$ _ $e_{60} =$ _ $e_0 =$ _

$$A^{ROT} = Q \cdot A \cdot Q^T = \begin{bmatrix} - & - \\ - & - \end{bmatrix} \qquad - = e_{60} = 400\cdot 10^{-6}$$

$$Q = \begin{bmatrix} \cos 60 & \sin 60 \\ -\sin 60 & \cos 60 \end{bmatrix} \quad A = \begin{bmatrix} \varepsilon_o & \varepsilon_{12} \\ \varepsilon_{12} & \varepsilon_{2} \end{bmatrix} \quad Q^T = \begin{bmatrix} \cos 60 & -\sin 60 \\ \sin 60 & \cos 60 \end{bmatrix}$$

poi ruoto x un altro angolo, una volta ottenuta $\Rightarrow$ calcolo $\sigma = (\lambda\, tr\varepsilon) I + 2\mu\varepsilon$

matrice def. $\varepsilon = \begin{bmatrix} - & - \\ - & - \end{bmatrix}$

$$\lambda = \frac{2\mu \cdot \nu}{1-2\nu} \qquad \mu = \frac{E}{2\cdot(1+\nu)}$$