

Strategia d'attacco

~~Formatori~~

I idealizzazione [~~Algoritmo~~ $\frac{R}{n}$; $\frac{R}{m}$; $\frac{R}{k}$; ...]

II classificazione struttura $\left\{ \begin{array}{l} \text{no?} \\ \text{iper?} \end{array} \right.$ [solo se vedi circuiti chiusi].

III $\left\{ \begin{array}{l} \text{no} \\ \text{iper} \end{array} \right.$ $\left\{ \begin{array}{l} \text{internate?} \\ \text{esternate?} \end{array} \right.$

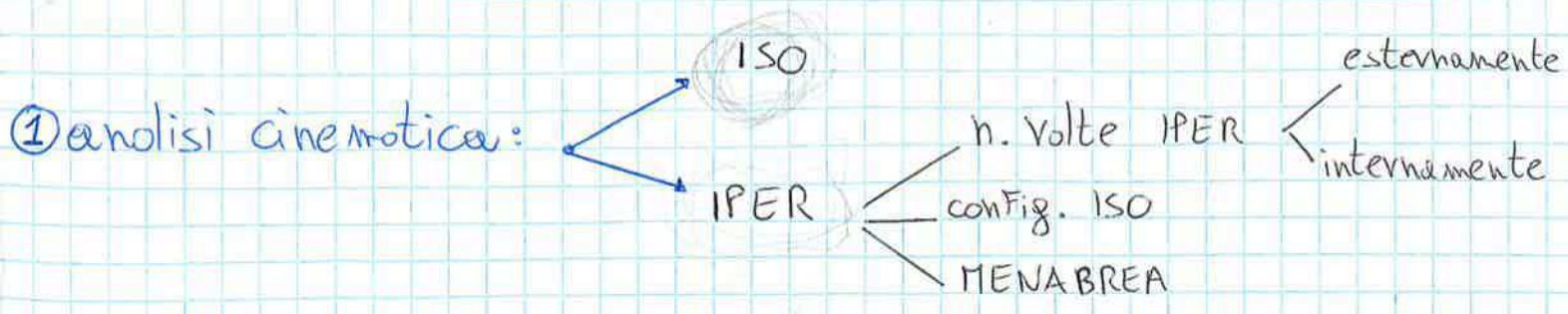
IV scegli le incognite iperstatiche [ENB! non solo B!]

V Risolvi la struttura no associata! [Prima risolvere esterne e poi esplodere!]

VI ~~Diagrammi~~ Membrane / P. f.v. Se chiede il calcolo

Diagrammi ~~di~~ ~~ip~~ ~~statici~~ \Rightarrow $\frac{dU}{dx} = 0 \Rightarrow \frac{dU}{dF} = \delta$

VII
VIII



MENABREA:

= en. elastico def.

$$\mathcal{E} = \frac{1}{2EI_x} \int_0^l M^2 dz + \frac{1}{2EA} \int_0^l N^2 dz + \frac{1}{2} \frac{F_{norm}^2}{K} + \frac{1}{2} \cdot \frac{M_{norm}^2}{K_{rot}} + \frac{1}{2GI_p} \int_0^l M_t^2 dz$$

$K = \frac{E \cdot A}{L}$ $G = \frac{E}{2(1+\nu)}$

1 volta IPER: $\frac{d\mathcal{E}}{dx} = 0 \dots X = \dots$

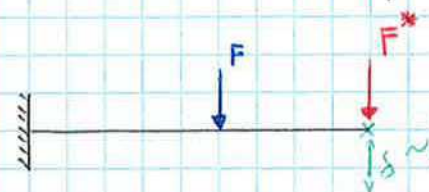
2 volte IPER: $\begin{cases} \frac{d\mathcal{E}}{dx} = 0 \dots X = \dots \\ \frac{d\mathcal{E}}{dy} = 0 \dots Y = \dots \end{cases}$

SPOSTAMENTI:

$\mathcal{E} = \text{come prima} \dots \delta = \frac{d\mathcal{E}}{dF}$



se:

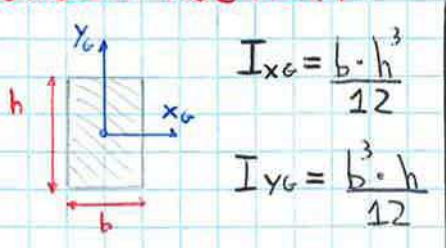


calcola spostamento

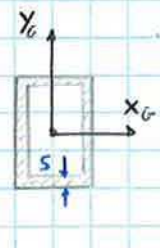
inserisco F* fittizia

$\mathcal{E} = \mathcal{E}(F; F^*) \rightarrow \frac{d\mathcal{E}}{dF^*} = \delta^* \rightarrow \lim_{F^* \rightarrow 0} \delta^* = \delta(F)$

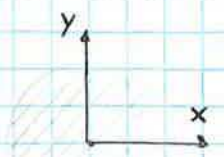
MOMENTI INERZIA:



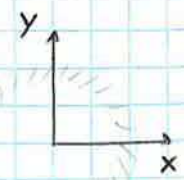
$I_{x0} = \frac{b \cdot h^3}{12}$
 $I_{y0} = \frac{b^3 \cdot h}{12}$



$I_{x0} = \frac{b \cdot h^3}{12} - \frac{(b-2s)(h-2s)^3}{12}$
 $I_{y0} = \frac{b^3 \cdot h}{12} - \frac{(b-2s)^3(h-2s)}{12}$

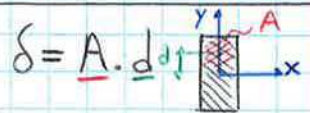


$I_x = I_y = \frac{\pi \cdot R^4}{4}$
 $I_p = 2I_x = 2I_y = \frac{\pi \cdot R^4}{2}$



$I_x = I_y = \frac{\pi}{4} [r_e^4 - r_i^4]$
 $I_p = \frac{\pi}{2} [r_e^4 - r_i^4]$

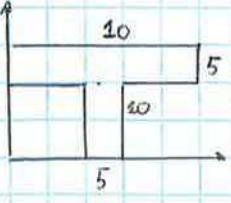
MOM. statico:



BARICENTRO: $X_G = \frac{\delta y}{A_{tot}}$

$Y_G = \frac{\delta x}{A_{tot}}$

esempio:

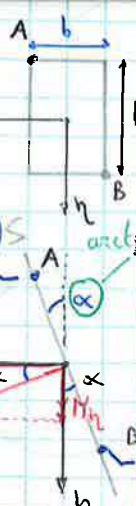


$X_G = \frac{\delta y}{A_{tot}} = \frac{(5 \cdot 10) \cdot 5 + (5 \cdot 10) \cdot 5}{5 \cdot 10 \cdot 2}$

$Y_G = \frac{\delta x}{A_{tot}} = \frac{(5 \cdot 10 \cdot 12,5) + (5 \cdot 10) \cdot 5}{5 \cdot 10 \cdot 2}$

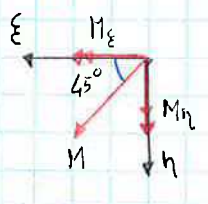
verifiche sveramento:

$$\sigma = \pm \frac{N}{A} \pm \frac{M_x \cdot h}{I_x} \pm \frac{M_y \cdot \epsilon}{I_y}$$

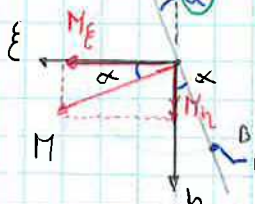


$$\sigma_{zA} = \pm \frac{M_x(-h/2)}{I_x} \pm \frac{M_y(b/2)}{I_y}$$

$$\sigma_{zB} = \pm \frac{M_x(-b/2)}{I_x} \pm \frac{M_y(-h/2)}{I_y}$$



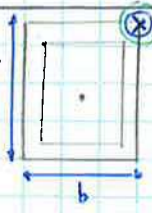
$$\sigma^+ = \sigma^- = \frac{M \cdot R}{I_x}$$



$$\sigma_{zA} = \pm \frac{M_x \cdot \tilde{h}}{I_x} \pm \frac{M_y \cdot \tilde{\epsilon}}{I_y}$$

$$\sigma_{zB} = \pm \frac{M_x \cdot \tilde{h}}{I_x} \pm \frac{M_y \cdot \tilde{\epsilon}}{I_y}$$

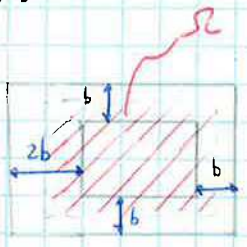
PRESSO-FLESSIONE:



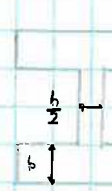
$$\sigma_{zA} = -\frac{N}{A} - \frac{M_x \cdot h}{I_x} - \frac{M_y \cdot b}{I_y}$$

tau generato da TORSIONE:

$$\tau = \frac{M_z \cdot R}{I_p}$$



$$\tau_{max} = \frac{M_T}{2R \cdot b_{min}}$$

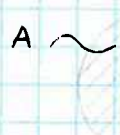


$$\tau_{max} = \frac{M_T}{I_T} \cdot b_{max}$$

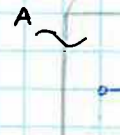
$$I_T = \sum i \cdot \omega_i \cdot b_i^3$$

tau generato da taglio:

$$\tau_{max} = \frac{T \cdot \delta_{max}}{I \cdot b}$$



$$\tau_G = \frac{4}{3} \frac{Q}{A}$$



$$\tau_G = \frac{3}{2} \frac{Q}{A}$$

VON MISES: $\sigma_{eq} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - (\sigma_x \sigma_y + \sigma_x \sigma_z + \sigma_y \sigma_z) + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)} \leq \frac{\sigma_{sy}}{\gamma_s}$

$$\sigma_{eq} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_I - \sigma_{III})^2}$$

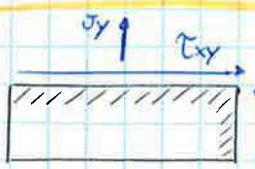
$$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2}$$

colli sicurezza $\gamma_s = \frac{\sigma_{sy}}{\tau_{eq}}$

TRESCA: $\sigma_{eq} = \sqrt{\sigma^2 + 4\tau^2}$

$$\sigma_{eq} = \max \left\{ |\sigma_I - \sigma_{II}|, |\sigma_I - \sigma_{III}|, |\sigma_{II} - \sigma_{III}| \right\} \leq \frac{\sigma_{sy}}{\gamma_s}$$

MOHR: convenzione:



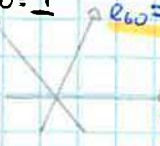
$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}$$

$$\sigma_{principali} = \det[\sigma - \lambda I]$$

$$\sigma_p = \frac{1}{2} \arctg \left[\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right]$$

graficamente: $P^I(\sigma_{xx}, \sigma_{xy})$ $P^{II}(\sigma_{yy}, \sigma_{xy})$

ROSETTA:



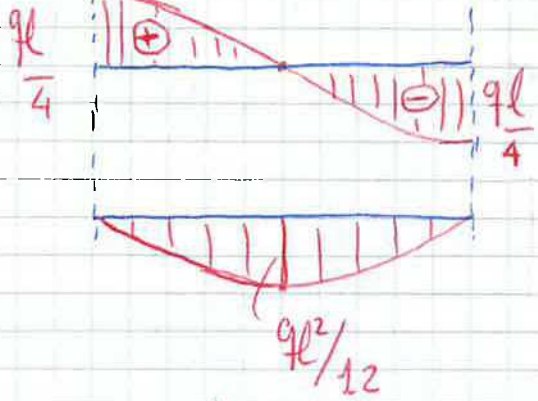
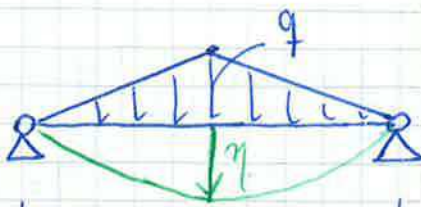
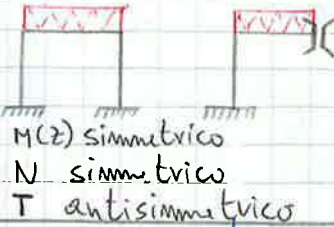
$$A = Q \cdot A \cdot Q^T = \begin{bmatrix} - & - \\ - & - \end{bmatrix}$$

$$= e_{60} = 400 \cdot 10^6$$

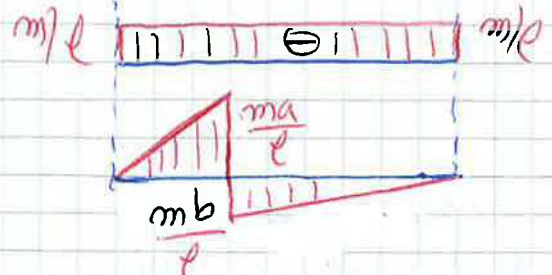
poi moltiplica x un altro angolo. un volta ottenute \Rightarrow calcola $\sigma = (\lambda \cdot tr \epsilon) I + 2\mu \epsilon$

matrice def. $\epsilon = \begin{bmatrix} - & - \\ - & - \end{bmatrix}$

STRUT. SIND. CARICATA SIND.

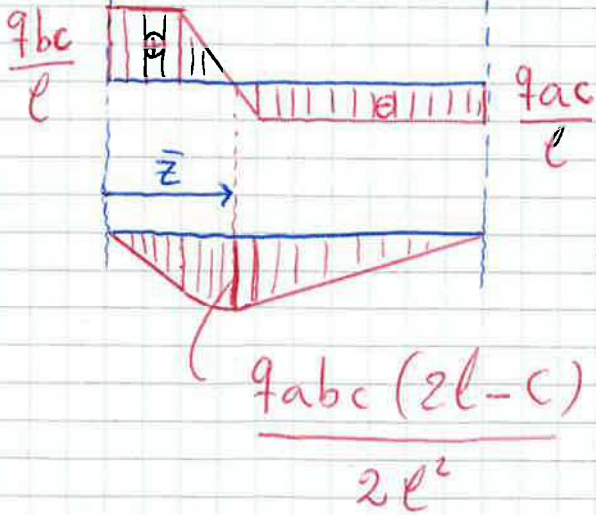
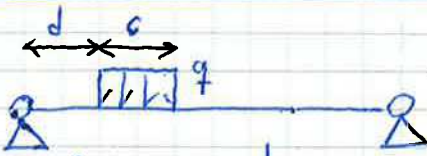


$$\eta_{elz} = \frac{ql^4}{120EI}$$

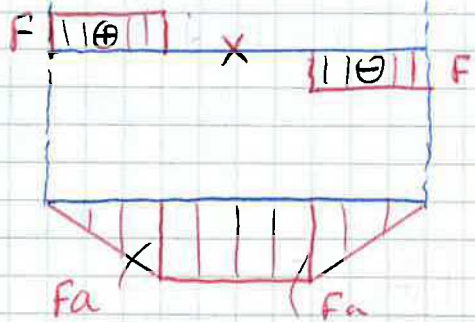
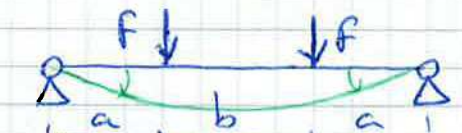
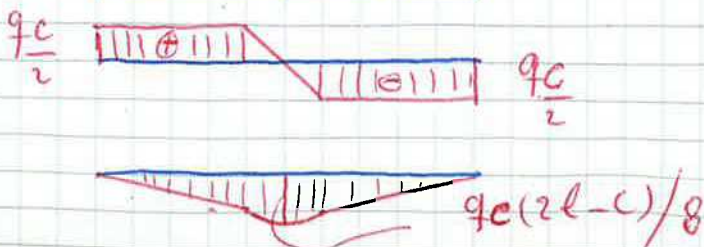
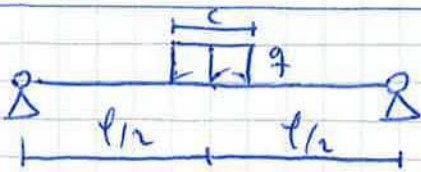


$$\varphi_A = \left[\frac{l^2 - 3b^2}{6EI} \right] m$$

$$\varphi_B = \left[\frac{l^2 - 3a^2}{6EI} \right] m$$

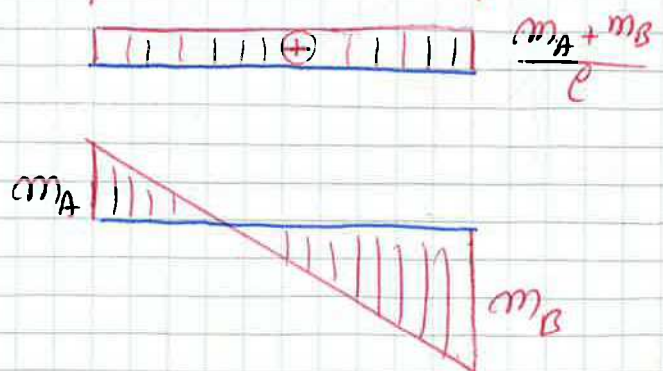
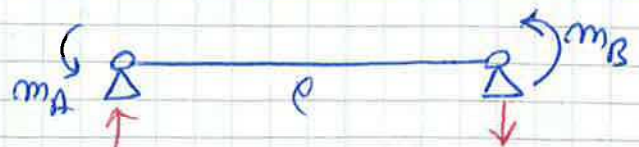


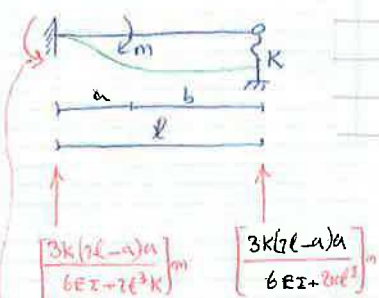
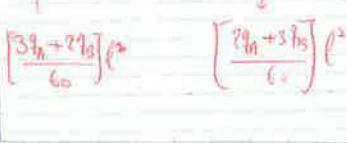
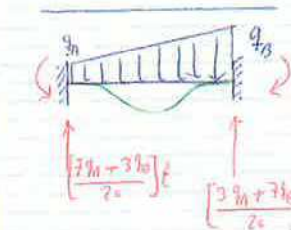
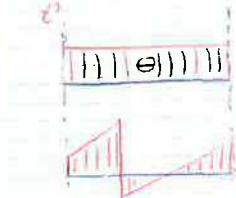
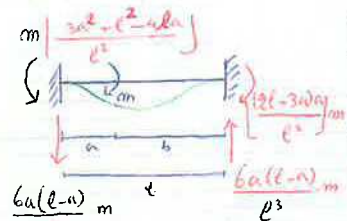
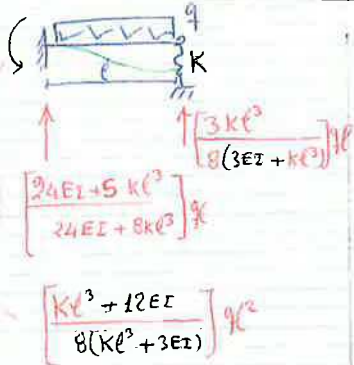
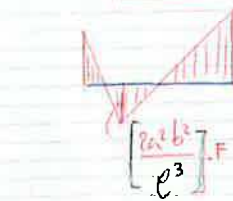
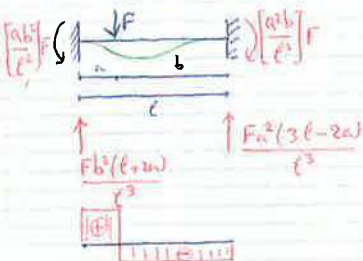
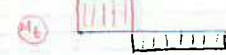
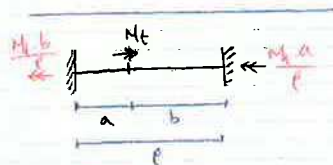
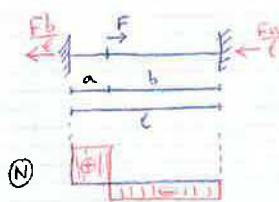
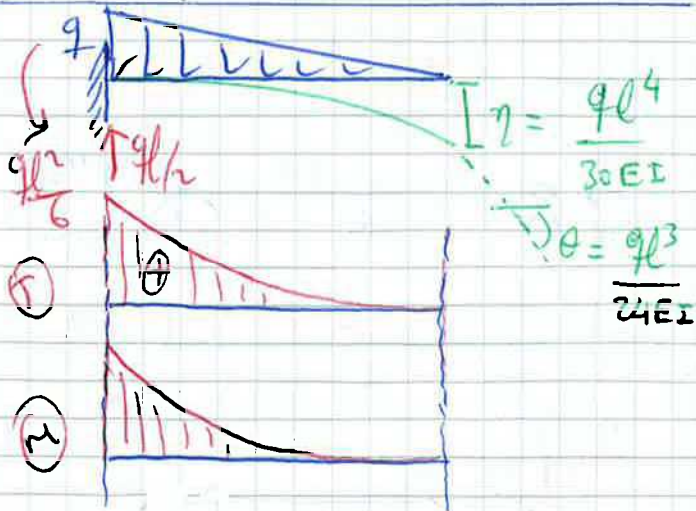
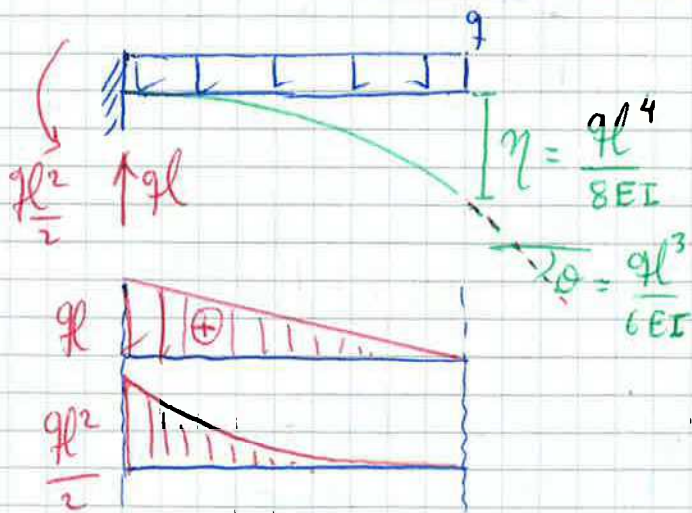
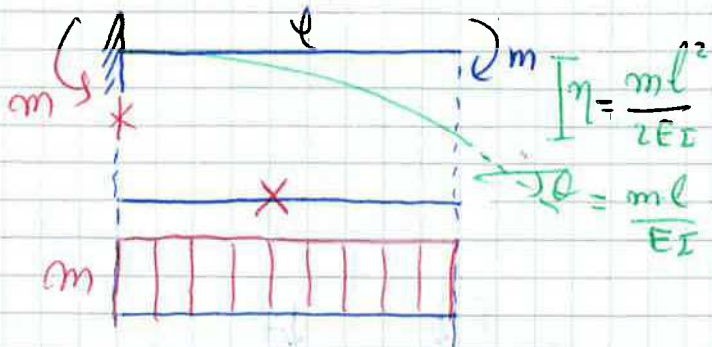
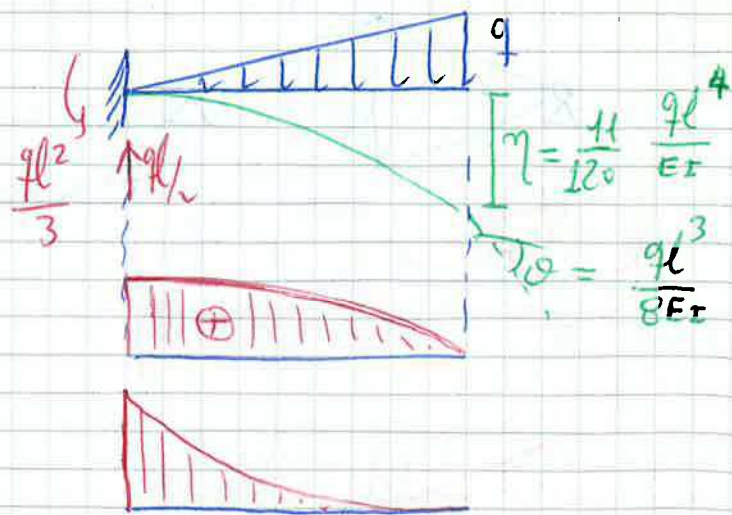
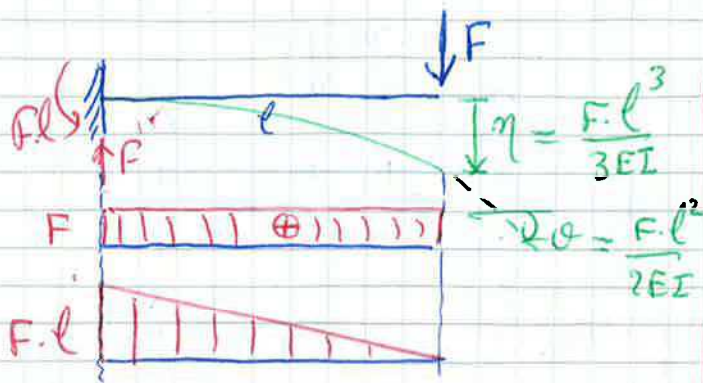
$$\bar{z} = d + \frac{qbc}{2l}$$



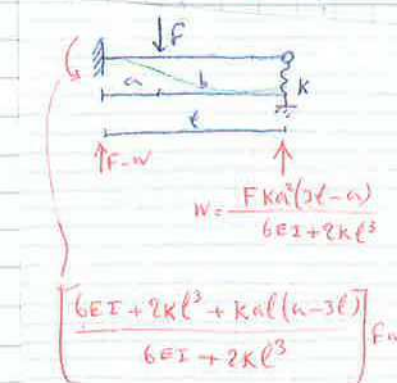
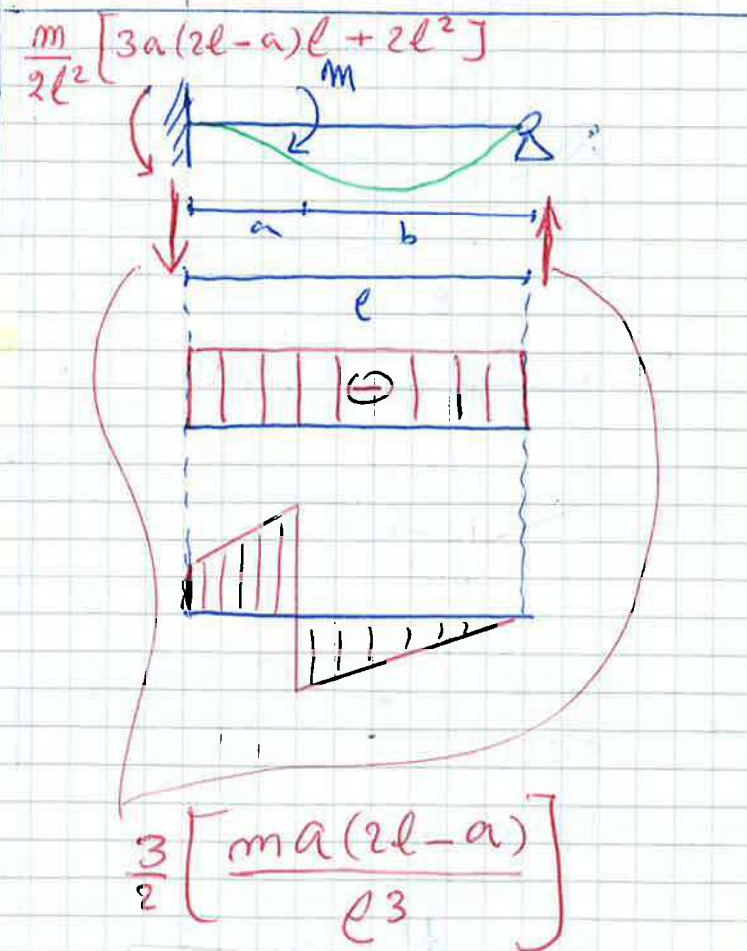
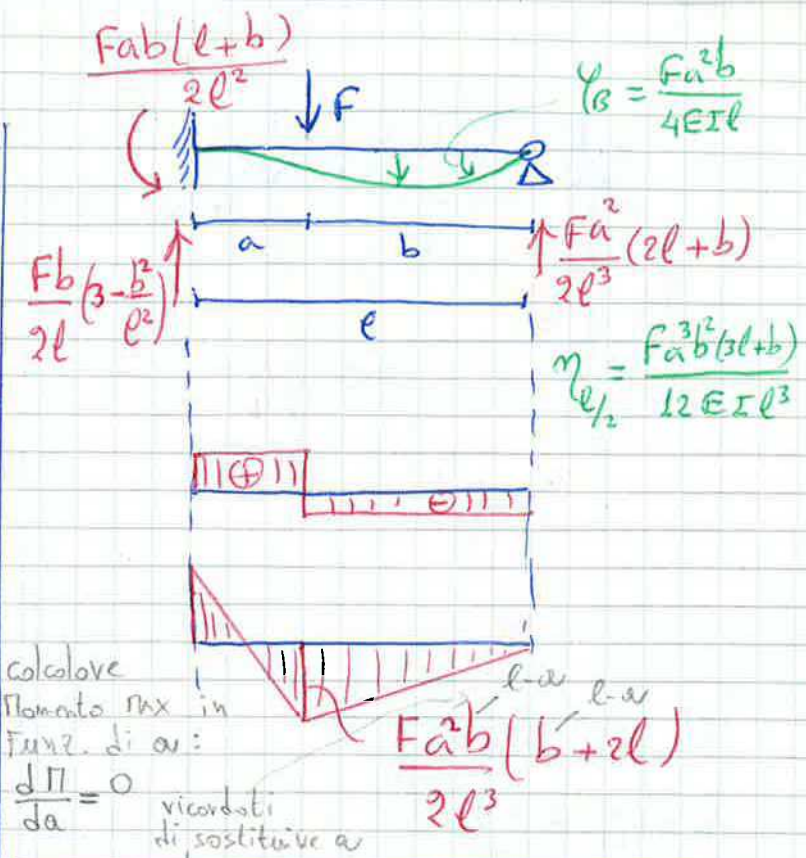
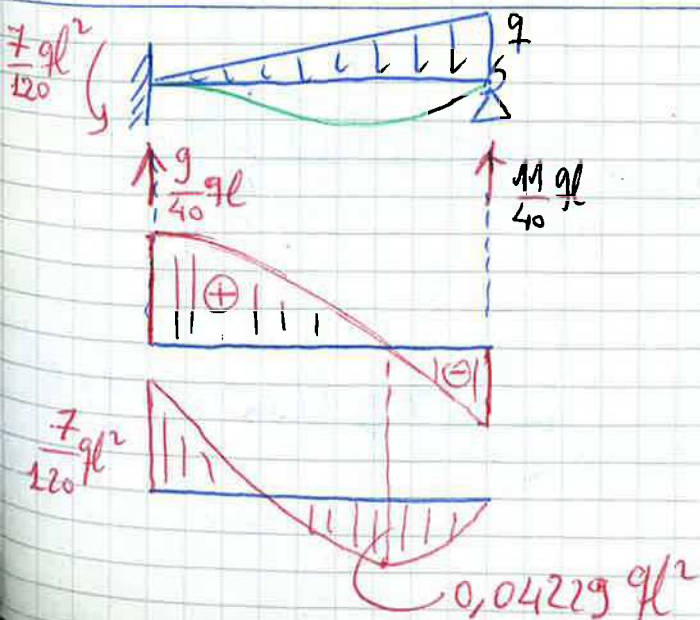
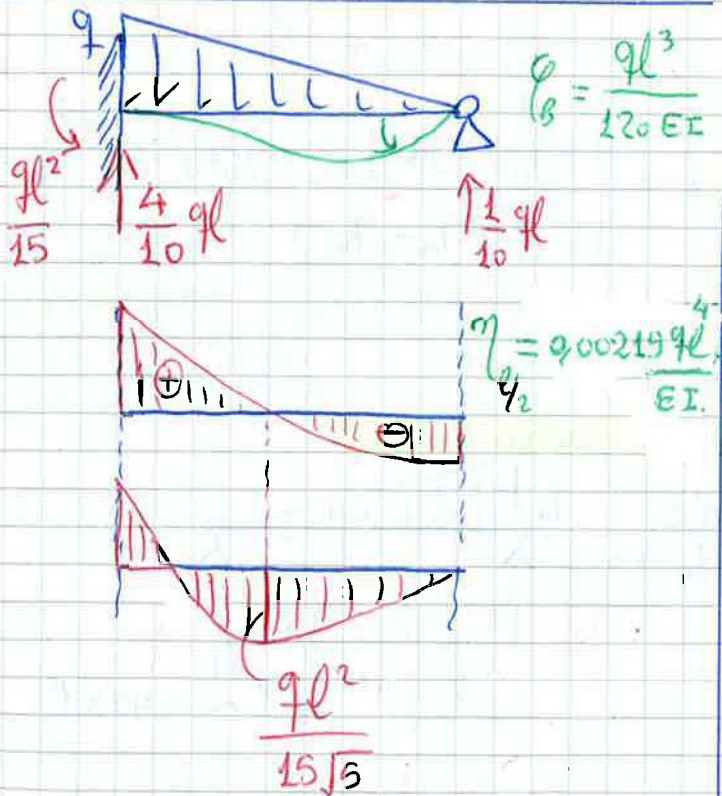
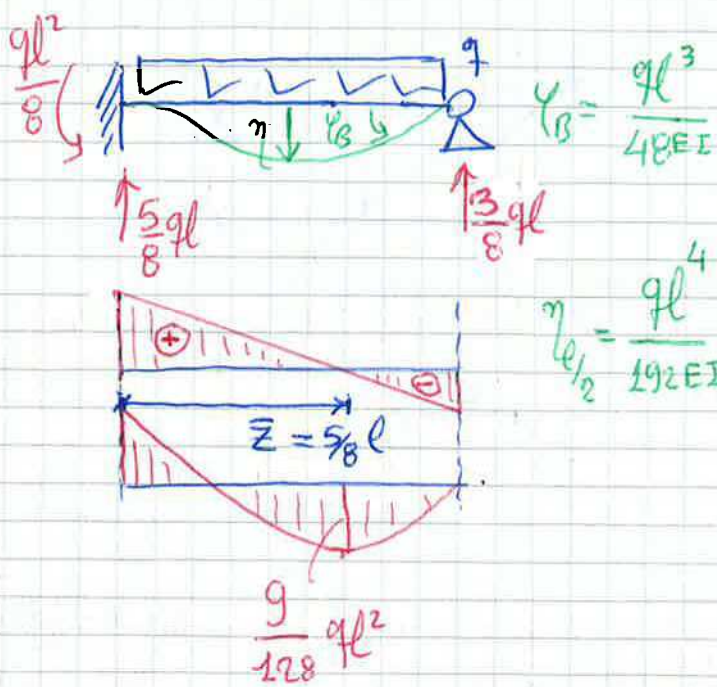
$$\varphi_A = -\varphi_B = \frac{Fa(l-a)}{12EI}$$

$$\eta_{elz} = \frac{Fa(-3l^2 - 4a^2)}{24EI}$$

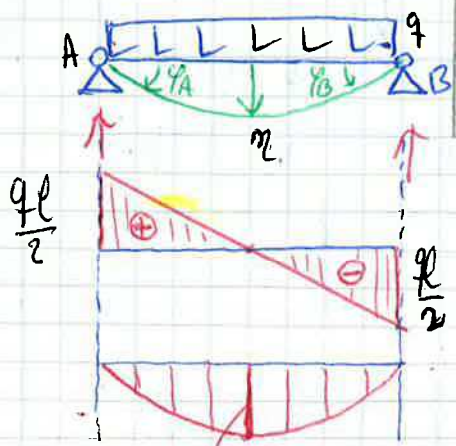
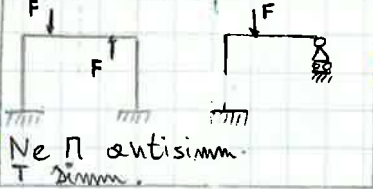




$$\frac{6EI + 2kl^2 + 3kla(a-l)}{6EI + 2kl^3}$$



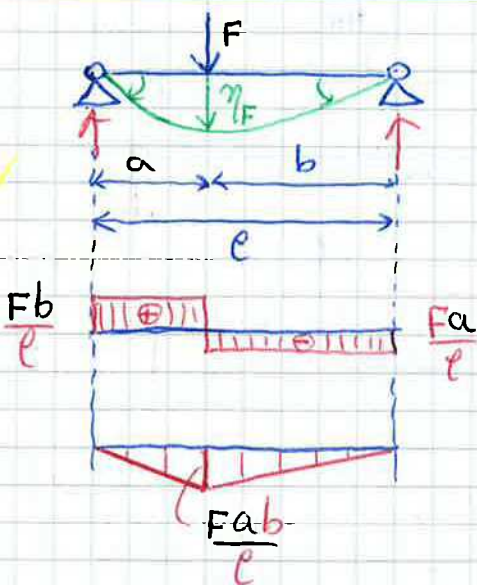
STRUT. SIMM. CARIC ANISIMM.



$$\eta_{q/2} = \eta_{max} = \frac{5}{384} \frac{ql^4}{EI}$$

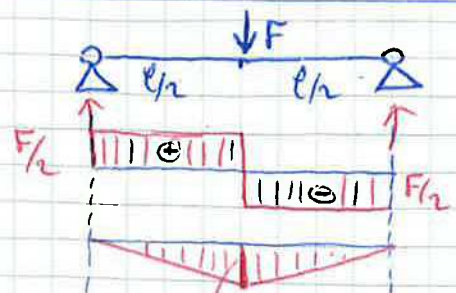
$$\varphi_A = -\varphi_B = \frac{1}{24} ql^3$$

$$\frac{ql^2}{8}$$

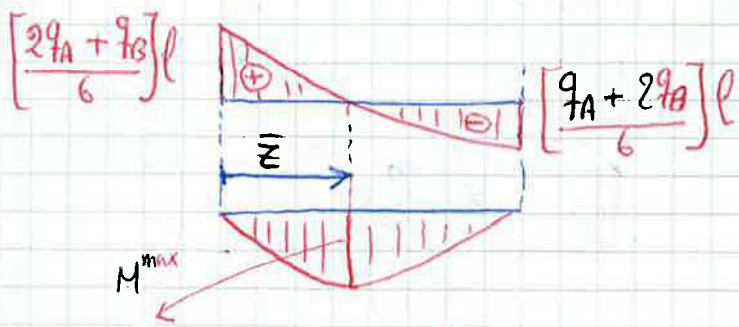
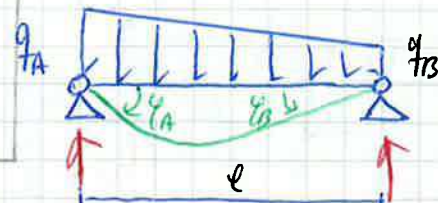


$$\eta_F = \frac{F a^2 b^2}{3EI l}, \quad \varphi_A = \frac{F a b (l+b)}{6EI l}$$

$$\varphi_B = \frac{F a b (l+a)}{6EI l}$$



$$\eta_{l/2} = \frac{F l^3}{48EI}, \quad \varphi_A = -\varphi_B = \frac{F l^2}{16EI}$$



$$\left[\frac{2q_A + q_B}{6} l \right]$$

$$\left[\frac{q_A + 2q_B}{6} l \right]$$

$$M^{max}$$

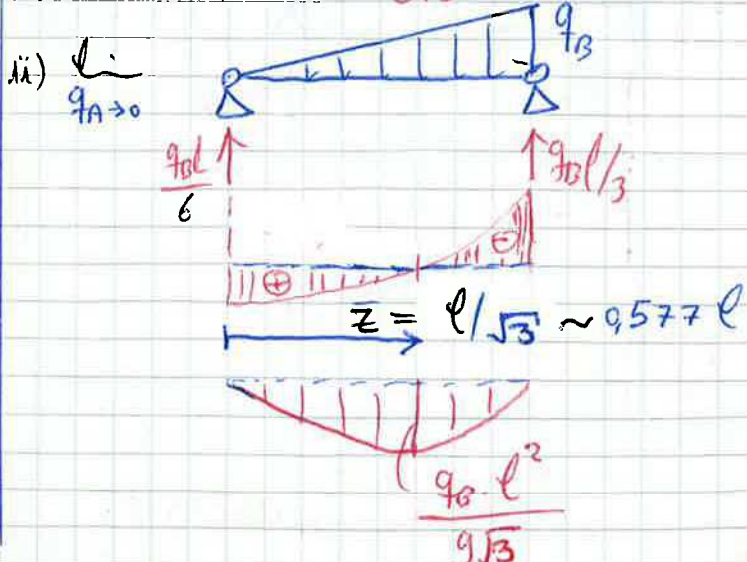
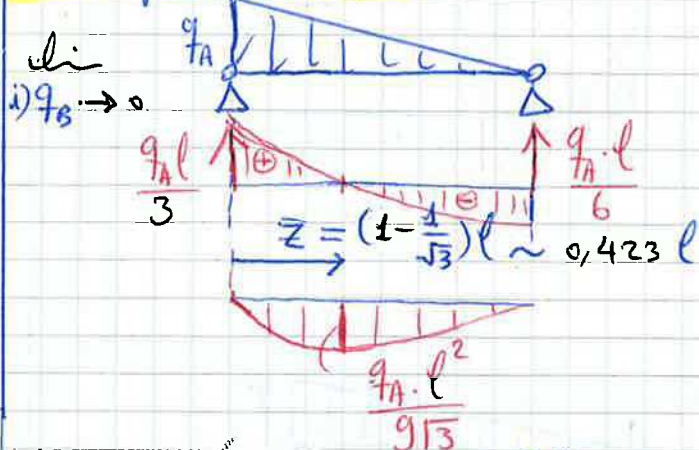
$$\left[\frac{2\sqrt{3}}{9} \left(\frac{q_A^2 + q_A q_B + q_B^2}{q_B^2} \right)^{3/2} - \frac{9 q_A (q_A + q_B)}{q_B^2} \right]$$

$$54 (q_A - q_B)^2 \cdot q_B^2$$

$$\bar{z} = \left[\frac{3q_A - \sqrt{3} \sqrt{q_A^2 + q_A q_B + q_B^2}}{3(q_A - q_B)} \right] l$$

$$\varphi_A = \frac{(8q_A + 7q_B) l^3}{360EI}, \quad \varphi_B = \frac{(7q_A + 8q_B) l^3}{360EI}$$

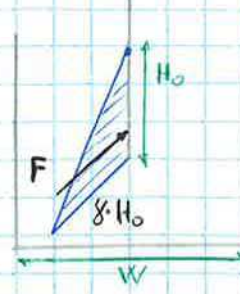
Casi particolari:



RUOTA DENTATA: $F_T = 2q \cdot l \cdot P = W$
 $\frac{F_T}{mm} = h \cdot D = mm$

PARATIA: con $P = 1000 \text{ kg/m}^3 \rightarrow \delta = P \cdot g$

FRESA:



$$F = \frac{1}{2} (\delta \cdot H \cdot W) \cdot H$$

base

$$M_T = \int_0^{\pi/2} q \cdot ds \cdot r = \int_0^{\pi/2} r^2 \cdot q \cdot d\theta = (r^2 \cdot q) \cdot \frac{\pi}{2} \Rightarrow q = \frac{2 M_T}{r^2 \cdot \pi}$$

$$F_x = \int_0^{\pi/2} q \cdot \sin\theta \cdot ds = -q \cdot r = -\frac{2 M_T}{r^2 \cdot \pi} \cdot r$$

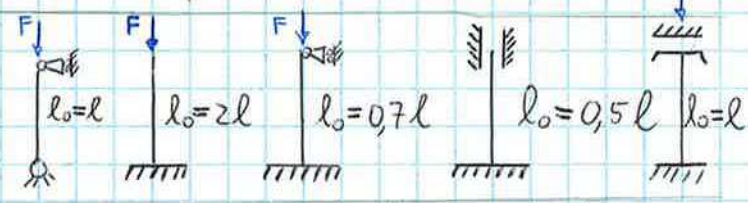
$$F_z = \int_0^{\pi/2} q \cdot \cos\theta \cdot ds = q \cdot r = +\frac{2 M_T}{r^2 \cdot \pi} \cdot r$$

VERIFICA INSTABILITÀ X COMPRESSIONE:

$$\frac{\pi^2 \cdot E \cdot I_{min}}{l_0^2} < N_{applicato}$$

altrimenti instabile
 prendere: $I < I_{y}$ o I_{x}

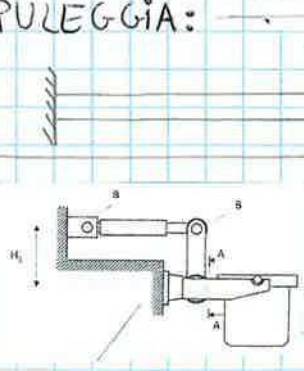
con l_0 :



ERRORE DI PONTAGGIO: $\delta = -$
 N.B. sdatamente serve meno forza per tirare la reticolare.

② pongo FORZA F nella struttura reticolare e visdov costigliano all'incontrovis $\frac{d\epsilon}{dF} = \delta$

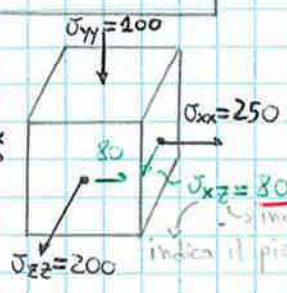
PULEGGIA:



CONSIDERARE (F) F_{max}
 $F_{cos\alpha}$

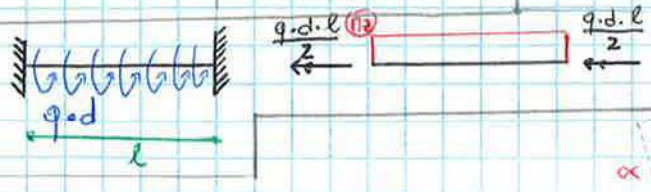
ES. MATRICE DI POHR:

$$S = \begin{bmatrix} 250 & 0 & -80 \\ 0 & -100 & 0 \\ -80 & 0 & 200 \end{bmatrix}$$

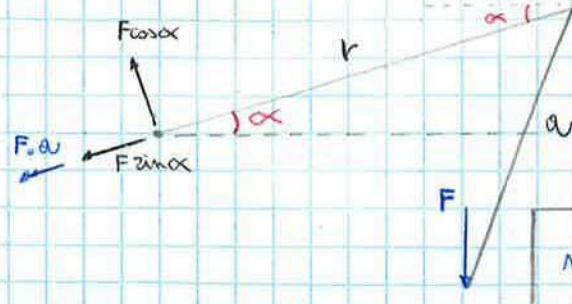
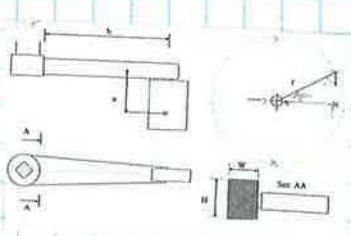


N.B. cambio segno \rightarrow X convenzione POHR.
 indica il piano dove si trova

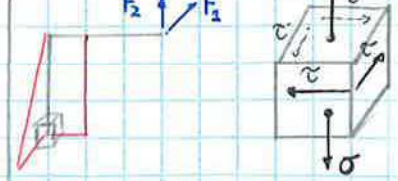
TORCENTE DISTRIBUITO:



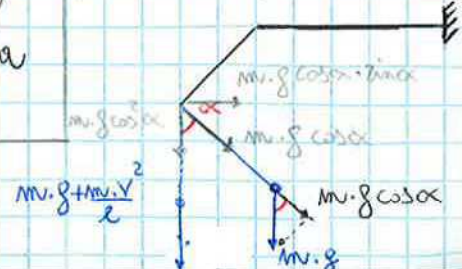
PEDIVELA:



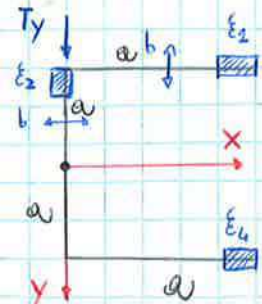
PULEGGIA:



PENDOLINO:



DOUBASKI:

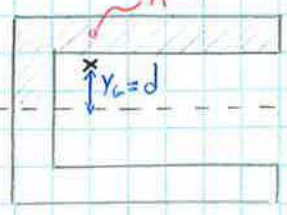


$$\delta_x(\xi_1) = (\xi_1 \cdot b) \cdot a \quad \left\{ \begin{array}{l} \xi_1 = 0 \dots \delta_x = 0 \\ \xi_1 = a \dots \delta_x = a^2 \cdot b \end{array} \right.$$

$$\delta_x(\xi_2) = -(\xi_2 \cdot b) \cdot \left(a - \frac{\xi_2}{2}\right) - a^2 \cdot b \quad \left\{ \begin{array}{l} \xi_2 = 0 \dots \delta_x = -a^2 \cdot b \\ \xi_2 = a \dots \delta_x = -\frac{3}{2} a^2 \cdot b \\ \xi_2 = 2a \dots \delta_x = -a^2 \cdot b \end{array} \right.$$

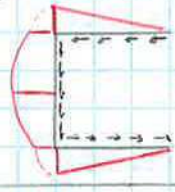
$$\delta_x(\xi_4) = \delta_x(\xi_1)$$

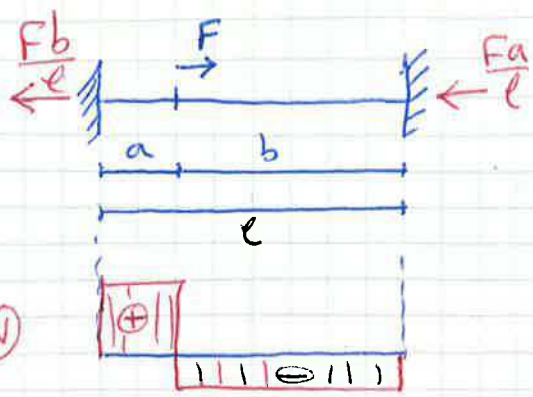
σ alternativa:



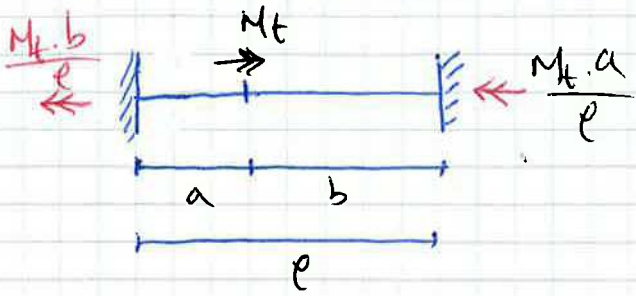
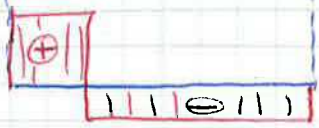
$$\delta_{max} = A^* \cdot d = A^* \cdot y_G$$

- δ_{max} sul baricentro
 - $\perp T \rightarrow \delta$ lineare
 - $\parallel T \rightarrow \delta$ parabolico

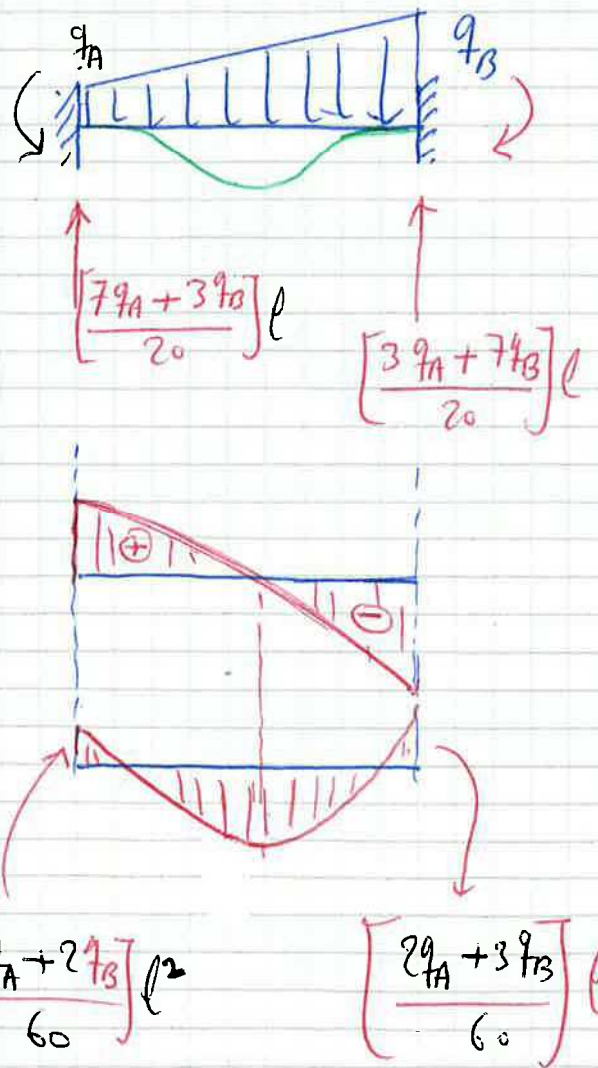
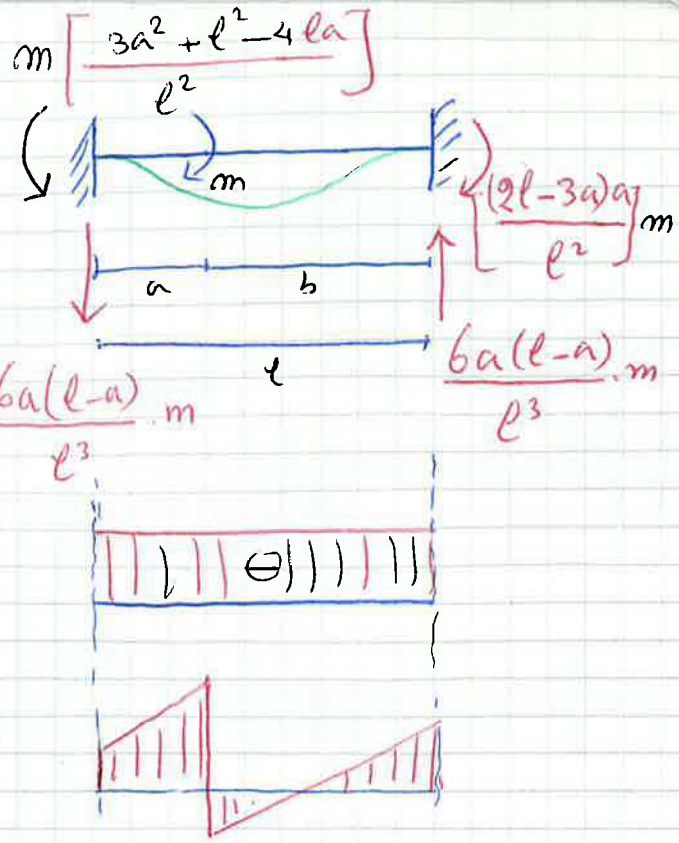
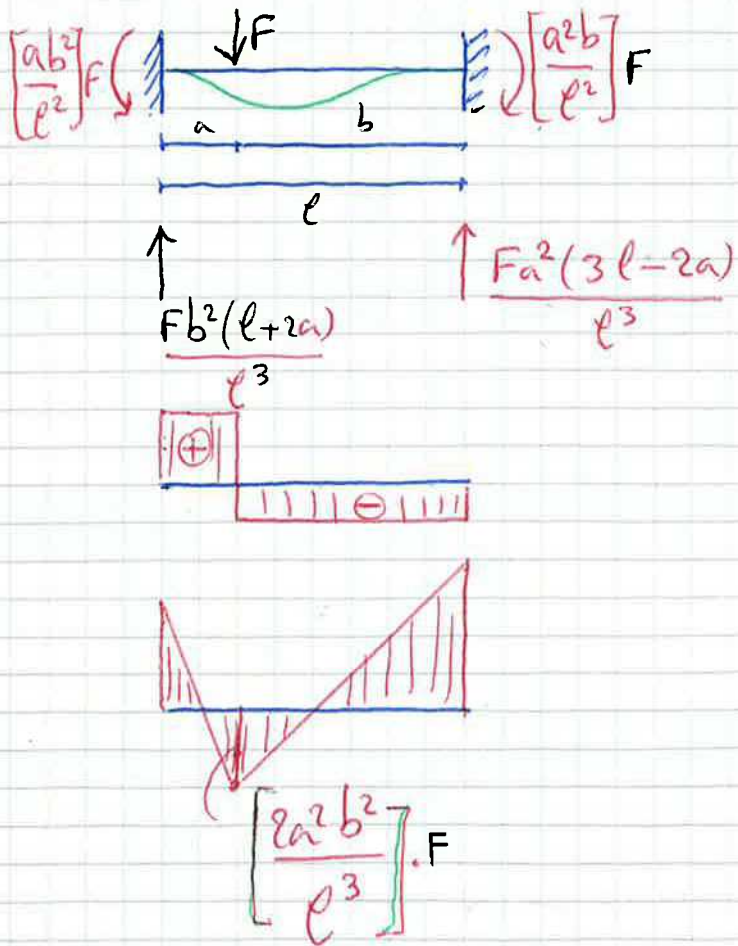
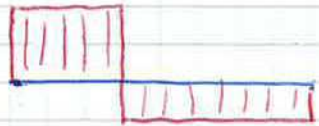


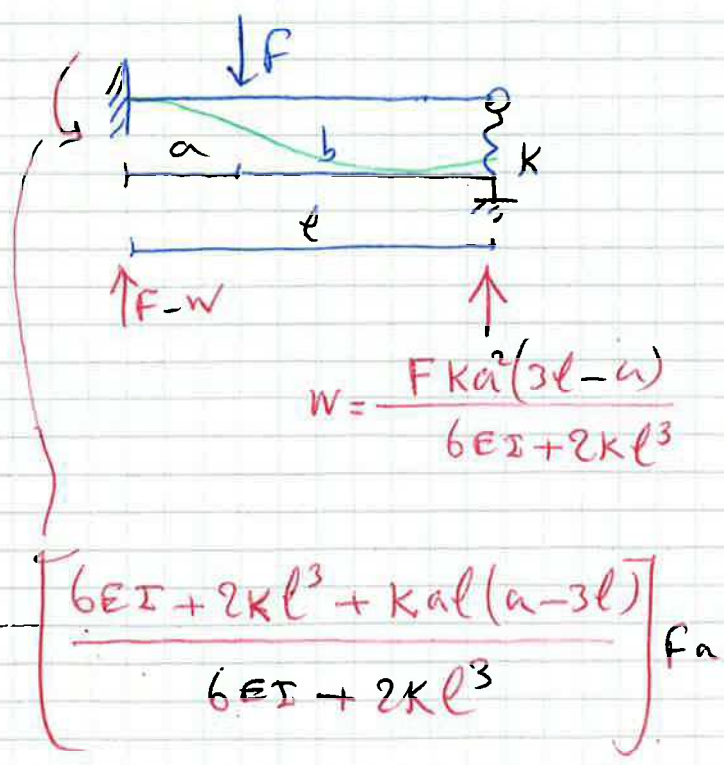
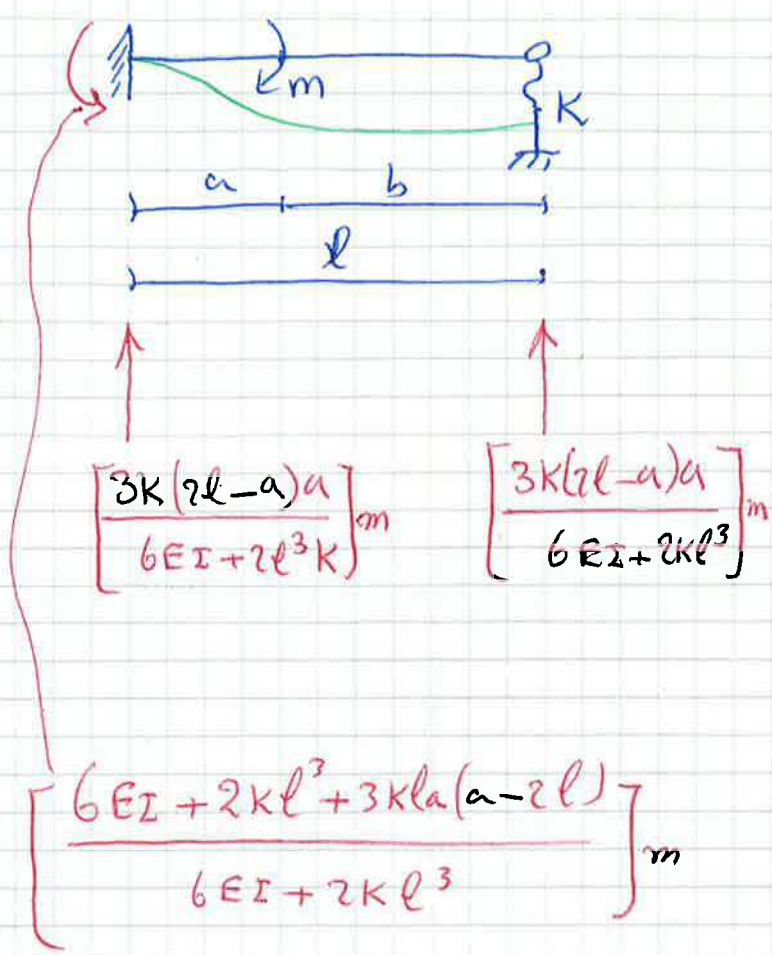
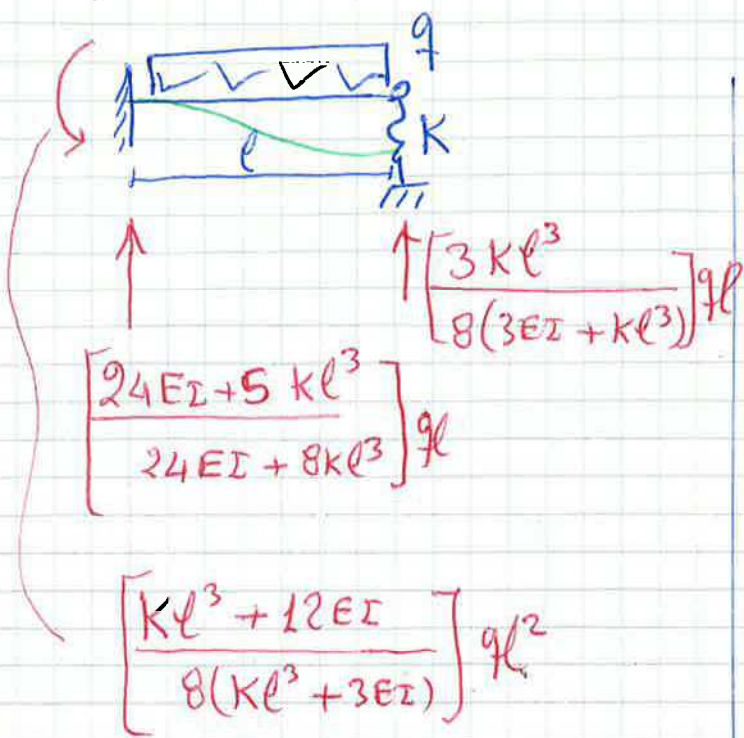


(N)



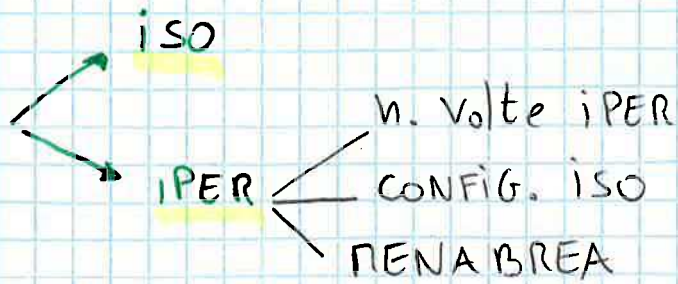
(M_t)





FORMULARIO:

① analisi cinematica



② menabrea:

energia elastica def. $\mathcal{E} = \frac{1}{2EI_x} \int_0^l M^2 dz + \frac{1}{2E \cdot A} \int_0^l N^2 dz + \frac{1}{2} \frac{F_{rot}^2}{K} + \frac{1}{2} \frac{\Pi_{rot}^2}{K_{rot}} + \frac{1}{2GI_t} \int_0^l \Pi_t^2 dz$

$\int_0^l \Pi_t^2 dz$ \rightarrow I_{polare}

se 1 volta IPER: $\frac{d\mathcal{E}}{dx} = 0 \quad \dots \quad x = \dots$

$G = \frac{E}{2(1+\nu)}$

se 2 volte IPER: $\begin{cases} \frac{d\mathcal{E}}{dx} = 0 \quad \dots \quad x = \dots \\ \frac{d\mathcal{E}}{dy} = 0 \quad \dots \quad y = \dots \end{cases}$

in seguito sostituire questi valori all'interno della struttura e trovare i diagrammi.

③ spostamenti:

energia elastica $\mathcal{E} =$ già calcolata, ma deve sostituire i valori trovati con Penabrea.



quando devo trovare la def. elastica. Mittente e ricevente

$\lim_{F^* \rightarrow 0} \delta^* = \delta(F)$

se Π_{torce} e $\Pi_{trazione}$

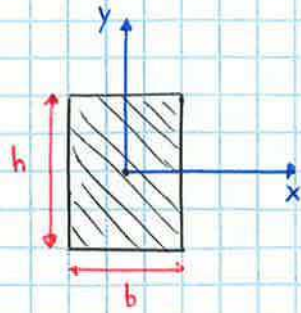
$\mathcal{E} = \int_0^l \Pi_t^2 dz \rightarrow \delta = \frac{d\mathcal{E}}{dF} = \dots$

$\delta_{tot} = \delta_{elast.} + \delta_{torc.}$

$\mathcal{E}_{torc.} = \int_0^l M_z^2 dz \rightarrow \delta = \frac{d\mathcal{E}}{dF} = \sigma \left(\frac{rod}{3} \right) \quad \dots \quad \delta_{torc.} = \theta \cdot l = \dots$

④ momenti di inerzia:

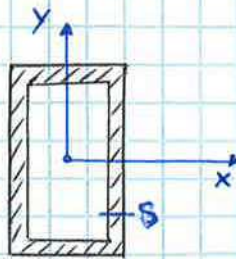
sez. rettangolare:



$$I_{xG} = \frac{b \cdot h^3}{12}$$

$$I_{yG} = \frac{b^3 \cdot h}{12}$$

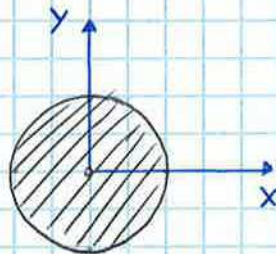
sez. rett. cava



$$I_{xG} = \frac{b \cdot h^3}{12} - \frac{(b-2s)(h-2s)^3}{12}$$

$$I_{yG} = \frac{b^3 \cdot h}{12} - \frac{(b-2s)^3 (h-2s)}{12}$$

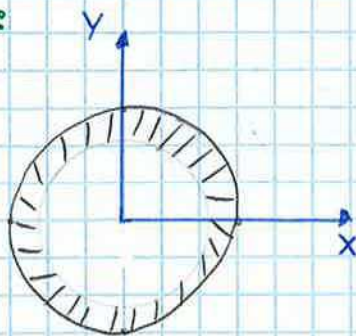
sez. circolare:



$$I_x = I_y = \frac{\pi \cdot R^4}{4}$$

$$I_p = 2 \cdot I_x = 2 I_y = \frac{\pi \cdot R^4}{2}$$

sez. circolare cava:



$$I_x = I_y = \frac{\pi}{4} [r_e^4 - r_i^4]$$

$$I_p = \frac{\pi [r_e^4 - r_i^4]}{2}$$

⑤ Verifica di snervamento:

- Innanzitutto, devo notare il punto più sollecitato guardando i diagrammi. In seguito individuo: σ e τ

$$\sigma = \frac{N}{A} \pm \frac{M_x \cdot y}{I_x} \pm \frac{M_y \cdot x}{I_y}$$

Formula generale σ

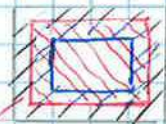
Compress. tra z

momento flettente

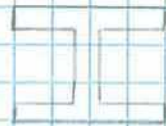
τ generata → TORSIONE:



$$\tau = \frac{M_z \cdot R}{I_p}$$



$$\tau = \frac{M_t}{2 \cdot b}$$

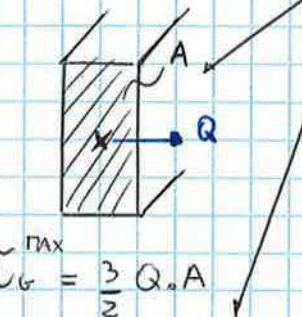


$$\tau = \frac{M_t \cdot b}{I_t}$$

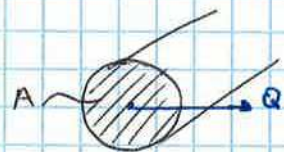
linea baricentrica

Taglio

momento statico



$$\tau_{G}^{\max} = \frac{3}{2} Q \cdot A$$



$$\tau_G = \frac{4}{3} \frac{Q}{A}$$

TAGLIO:

$$\tau_{\max} = \frac{T \cdot S}{I \cdot b}$$

$$S = A \cdot d$$

momento d'inerzia

lunghezza faccia

τ_{\max} sempre sul BARICENTRO

VON MISES:

$$\sigma_{eq} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - (\sigma_x \sigma_y + \sigma_x \sigma_z + \sigma_y \sigma_z) + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)}$$

cotensione

$$\sigma_{eq} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_I - \sigma_{III})^2}$$

Principali

$$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2}$$

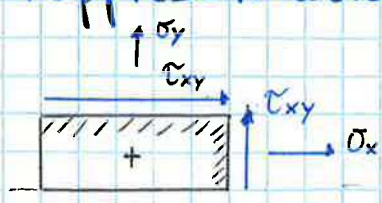
bidimensionali

TRESCA:

$$\sigma_{eq} = \sqrt{\sigma^2 + 4\tau^2}$$

TRESCA: $\tau_{max} = \max \left\{ \frac{|\sigma_I - \sigma_{II}|}{2}, \frac{|\sigma_I - \sigma_{III}|}{2}, \frac{|\sigma_{II} - \sigma_{III}|}{2} \right\} < \sigma_{ys}$
 principali

⑥ rappresentazione mohr:



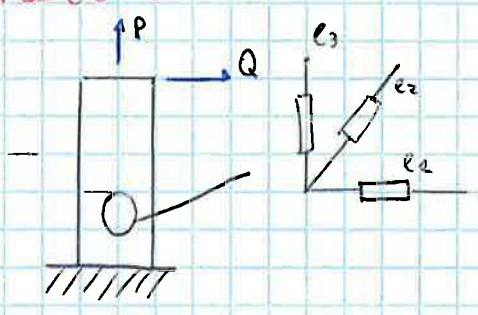
$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}$$

SFORZI PRINCIPALI: $\det[\sigma - \lambda I] \rightarrow \lambda_1 = \sigma_1$
 $\rightarrow \lambda_2 = \sigma_2$

$$\sigma_p = \frac{1}{2} \sigma_{xx} \pm \frac{1}{2} \sqrt{\sigma_{xx}^2 - 4\sigma_{xy}^2}$$

rappresentazione grafica: $p^I(\sigma_x; \sigma_{xy})$ $p^{II}(\sigma_y; \sigma_{xy})$

rosetta:



per trovare e_2 vogliamo di 45°

$$A^{ROT} = Q^T \cdot A \cdot Q$$

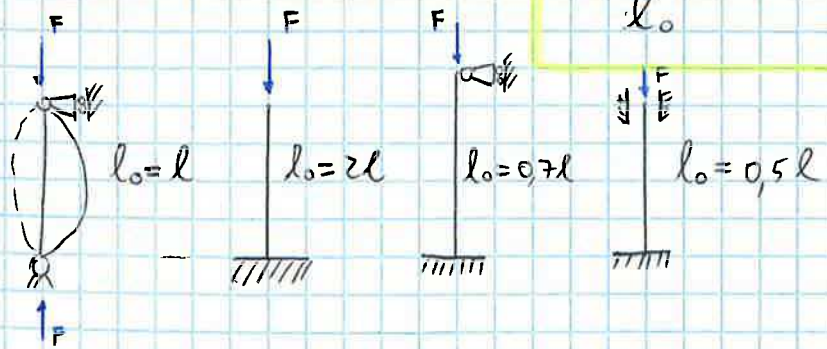
$$Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \dots & \dots & \dots \\ \dots & \dots & \epsilon_{zz} \end{bmatrix}$$

⑦ verifica instabilità:

per bielle compresse:

$$\frac{\pi^2 \cdot E \cdot I}{l_0^2} < N_{applicato} \rightarrow \text{altrimenti INSTABILE!}$$



per bielle in trazione:

$$\sigma = \frac{N}{A} < \sigma_{snervamento}$$